Looking beyond vision: Supports for students who are blind or visually impaired in mathematics

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1. Abstract

This review examines research on math achievement in students who are blind or visually impaired (BVI), and seeks to show the opportunities that BVIs have access to in order to demonstrate their knowledge of mathematics as well as the unique challenges they face and the ways in which these barriers have (or have not) been addressed. Math achievement for students who are BVI often lags that of their peers (Bell & Silverman, 2019; Cryer et al., 2013) though there is substantial evidence that vision is not required for the development of number sense or other math skills and concepts, and that students who are BVI can develop these skills using non-visual pathways (Amalric, 2018; Crollen & Collignon, 2020; Ahlberg & Csocsán (1999). However, few studies have explored math education for BVI students, and the limited research that exists on online math access for BVIs focuses on presentation modalities such as auditory methods, haptic feedback, and tactile input for the purpose of describing visual information (Scalise et al., 2018). Nevertheless, research indicates that given proper academic supports within the school setting (Giesen et al., 2012), and access to multi-representational materials (Scalise et al., 2018) that engage multiple senses (Crollen & Collignon, 2020), many of the barriers that prevent BVIs' access to mathematical content can be greatly reduced. With an inclusive assessment showing growth over time (Rottmann et al., 2020), and when given opportunities to engage independently (Beal & Rosenblum, 2018), BVIs can demonstrate their mathematical knowledge, perceive their own growth in mathematics, and gain some much-needed confidence in their ability to succeed in technical fields of education.

2. Introduction

2.1. Math achievement for students who are blind or visually impaired

Students who are blind or visually impaired (BVIs) are falling behind in mathematics classes (Bell & Silverman, 2019; Cryer et al., 2013), and as they get older, the gap between them and their non-disabled peers progressively widens (Giesen et al., 2012). With regard to achievement, 75% of BVI students are at least one grade behind and 20% are four or more grades behind (Gulley et al., 2017). This review of the literature seeks to showcase the opportunities that BVIs have access to in order to demonstrate their knowledge of mathematics as well as the unique challenges they face and the ways in which these barriers have (or have not) been addressed.

Despite the suboptimal achievement statistics cited above, there is evidence to suggest that limited vision does not affect BVIs' ability to develop strong math skills and concepts. Amalric (2018) found that the abstract and modal systems of the brain lit up when two blind mathematicians were considering the concepts of number and space, but not the visual cortex as might be expected. They concluded that brain networks needed for advanced mathematical thinking can develop in the absence of visual experiences. Before students get to the advanced part of learning mathematics, however, a strong understanding of number sense is needed (Ahlberg & Csocsán, 1999; Rottmann et al., 2020). There is a plethora of data showing that achievement in number sense tasks is linked to visual processes in the brain (Anobile et al., 2013; Dakin et al., 2011; Stoianov & Zorzi, 2012; Zhou et al., 2015). Nevertheless, regions responsible for working memory, tactile manipulation, and language lit up instead in the neural networks of individuals who were blind when they were performing basic number sense tasks

(Crollen & Collignon, 2020). In this study, the authors found that BVIs were equal to and one individual even out-performed people with normal levels of vision in number sense tasks. When studying how 25 Hungarian children between the ages of 5 and 9 who were born blind (with light perception or less) experience numbers, Ahlberg & Csocsán (1999) discovered that BVIs do not use their fingers to explore numbers like children with sight generally do, but rather, start their understanding by knowing numbers as words first; their fingers are primarily used for exploration of the world around them. Altogether, these studies indicate that vision is not mandatory for the development of number sense and imply that there is much potential for acquiring mathematical skills without the need to rely chiefly on visual processes. If the difficulty in learning mathematics for BVIs is not greatly restricted by limited vision, then perhaps it has more to do with the way in which information is presented, and how consistent that presentation is across all environments where BVIs interact with math content.

Giesen et al. (2012) shed some light on this idea. In a six-year longitudinal study, they discovered that BVIs in schools with more academic supports consistently out-performed on standardized tests for mathematics than those in schools lacking such robust support systems. Though these results are promising, there have been very few quality research studies published regarding BVIs and mathematics education. Blindness and visual impairment are considered to be a low incidence disability, and this has likely been a factor contributing to this lack of research. Klingenberg et al. (2019) conducted a systematic analysis of research between the years of 2000 and 2017 and found only 11 studies of moderate quality in relation to mathematics education for BVI students. Specifically with regard to digital learning for BVIs, a second meta-analysis of studies published within the same timeframe found that there is a “lack of scientific evidence to establish research-based practice related to learning programs for students with VI.” (Klingenberg et al., 2020, p. 51). Very few studies in either report described the inclusion criteria or the level of mathematical skills present in participants prior to the outset. Most studies also had small numbers of participants with wide varieties of ages and eye conditions. With regard to digital testing, most research has been focused on presentation modalities such as auditory methods, haptic feedback, and tactile input (Scalise et al., 2018) for the purpose of describing visual information. The authors concluded that these methods were not only time consuming for students, but were also only partially successful at eliminating access barriers to mathematical information.

2.2. Barriers to accessing math content

To better understand potential barriers BVIs experience when interacting with math content, it is helpful to consider some specific challenges. One challenge is that math and science related content for BVIs is often late, absent, or portrayed in a non-straightforward manner. About 85% of middle and high school BVI students surveyed reported that their science, technology, engineering, and mathematics (STEM) materials arrived late at least some of the time (Bell & Silverman, 2019). Further, a study assessing the quality of the production of braille and tactile math and science worksheets (Herzberg & Rosenblum, 2014) found that only 5% of worksheets given to students were totally without error and 20% had five or more errors. Moreover, when 10 teachers of the visually impaired were asked to describe mathematics graphics to BVIs according to the National Council of Accessible Materials’ guidelines, Rosenblum et al. (2020) discovered that there was wide variability in how teachers described the graphics, and in how they questioned students about the graphics features and the information presented.

The second factor presenting challenges to BVIs, according to Cryer et al. (2013), is the heavy reliance in STEM fields on visual representations and the difficulty in relaying the information from these resources to BVIs. Fitzpatrick, et al. (2018) noted that people with sight have access to rich semantic and syntactic information from visual representations in two-dimensional space that BVIs, because braille, large print, and audio are linear in nature, do not have. They expounded on this by saying that visual information is non-temporal and hence a person with sight can refer to it as a form of external information. Because of sound's inherent temporal nature, BVIs must employ a greater cognitive load to retain and manipulate technical information. In other words, representations in two dimensions provide a persistent visual cue so that people with sight can quickly and easily refer back to them, whereas linear representations are missing such cues. As a result, people with BVI, in order to intake the same information, must hold more in their memory and thus their cognitive load inevitably increases. Though BVIs demonstrate high levels of achievement in memory tasks since they must employ various strategies to compensate for missing visual details (Spinczyk, et al, 2019), maintaining concentration while processing sensory information presents a unique challenge to BVIs when accessing technical materials.

Karshmer and Farsi (2007) reported on five general modes of access that BVIs employ to acquire information from math materials. These approaches are: tactile, verbal (including screen reader software), tonal (musical note representations of graphs and diagrams), haptic, and an integrated approach that consists of more than one mode. According to Cryer et al. (2013), dealing with tactile materials while manipulating the elements of a math problem can cause more difficulties than the math content itself. Students who use large print materials experience similar issues because they can only see parts of the problem at one time. Hence, most of the research conducted regarding access to mathematics for BVIs has been focused on the verbal mode.

2.3. Auditory modes of access

Isaacson et al. (2010) studied how accurately students identified math expressions by listening to spoken content based upon the Math Speak guidelines developed by Abraham Nemeth (a blind mathematician) using different forms of the same expression. The authors found that those who heard expressions using Math Speak performed significantly better than those who heard other auditory forms of the same expression. Though these are promising results, it is important to note that the participants of this study were graduate students, and none had a visual impairment. A qualitative study using MathSpeak that did include participants who were visually impaired was conducted by Bouck and Weng in 2014. They considered the effectiveness of a supported algebra etextbook in two classrooms at a state school for the blind. A supported etextbook was defined as a textbook with additional accommodations such as Zoom speech to text and braille capabilities. The speech portion used MathSpeak to voice the problems to students, and students could choose the level of navigation to interact with text, e.g., by line, word, character, etc. The results indicated that students were confused about how and when to use the textbook, and why they would even bother using it in the first place: "I cannot communicate with it and ask questions" (Bouck and Weng, 2014, p. 135). In particular, students reported that the keyboard shortcuts were different than those of their native screen reading software, and those who did not read braille reported being more confused about the way in which auditory problems were voiced. In contrast, students felt motivated to use an iPad app to solve pre-algebra word problems (Beal & Rosenblum, 2018), and even performed slightly better with the app than with their traditional methods. When students used Microsoft Word in combination with MathPlayer (Frankel et al., 2017), a software program using the ClearSpeech protocol to voice math problems, they performed similarly to those using traditional methods. This indicates that both student performance and motivation tend to be similar to or exceed that of traditional methods when the technology is familiar to BVIs.

2.3.1. Cognitive load

With auditory technology, though, there are concerns that listening to mathematics problems will result in increased cognitive load. Da Paixão Silva et al. (2017) explored the amount of time and effort it took BVI users to ascertain specific information about the quadratic equation $x^2 + 4x + 4$ on a computer using the keystroke level model task analysis developed by Card, Moran, and Newell (1980). Using only keyboard commands in combination with a screen reading computer program (JAWS, NVDA, or ChromeVox), participants were tasked to find out if this equation had two identical real roots, if it opened up or down, and if it was complete. The authors' model equated a single keystroke to 0.28 seconds, a mental operation to 1.2 seconds, and the wait time for the screen reader to finish speaking at 1 second. JAWS had the fastest time recorded at 132 seconds, with 30 keystrokes, 28 mental operations, and 88 seconds of wait time. NVDA had the slowest, with 203.28 seconds, 21 keystrokes, 67 mental operations, and 117 seconds of wait time. The authors concluded that JAWS users required the least amount of time for the problem because of the small number of keystrokes required as well as the ability to navigate the equation in chunks. While NVDA users also had a small number of keystrokes, the entire equation was always read aloud, no matter what keystroke was used. It is important to note here that the participants in this study were chosen for their familiarity with the math concepts and so the amount of time taken was not correlated to lack of understanding.

Cryer (2013) gave an example of how mental operations can be affected by auditory input. If one were to say, "a plus b over c," that could have two meanings: $\frac{a+b}{c}$, or $a + \frac{b}{c}$. Researchers involved in the Logan Project (Gulley et al., 2017) developed a method known as process driven math that specifically addressed cognitive load difficulties for BVIs who access math primarily through their sense of hearing. Instead of voicing a math problem in one long chunk of characters, the human reader/scribe would instead break down a math problem into pieces according to its mathematical vocabulary. For example, the expression $y = \frac{x^2+2x-15}{x^2-7x+12}$, would be read as "y equals rational." Then, the student could dig deeper by asking, "what is the numerator of the rational?" The reader/scribe would then answer, "term plus term minus constant," which is indicative of the language that a student would hear in a general education course. This pattern of digging down into each aspect of the expression would continue until the math problem was simplified. Though process driven math involves a lot of training for both the student and the reader/scribe, Gulley et al. (2017) stated that developing software that operates in a similar manner would eliminate the need for a human scribe and give BVIs more autonomy when solving math problems.

2.3.2. Tonal and haptic modes of access

Tonal and haptic representations of math content are time consuming to create and are not as widely used as verbal and tactile representations (Karshmer & Farsi, 2007). This is because they are expensive to make and do not relay the same information as visual representations. Mathematics images become less effective as expressions become more complex (Bouck and Weng 2014). Nevertheless, if haptic and tonal feedback could be included in products that are widely available to everyone and are commonly used in classrooms everywhere, then some of the concerns regarding cost and time could be mitigated. For instance, Hahn et al. (2019) used a Samsung multimodal tablet in combination with Google Classroom to ask 22 BVIs questions about graphics. Some students were provided with the tablet, which had tonal and vibration feedback, while others were given tactile representations of the same graphics. The authors found that there was no significant difference in the number of correct answers in either group. While this might seem to suggest that it would not matter which method is used for graphics display for BVIs, the authors posit that training on the tablet would take far less time than learning to read braille and interpret tactile materials.

2.3.3. A multi-sensory approach

While all of these approaches have merit, the truly successful ones tend to provide a variety of interfaces for access (Karshmer & Farsi, 2007). Preliminary results from a study looking at the effectiveness of auditory image descriptions on math, science, and English language learning assessments for students with print disabilities (Ferrell et al., 2017) showed that braille readers who had access to both the voiced auditory descriptions and tactile graphics increased their likelihood of answering items correctly; braille users who only had access to tactile images actually had a decreased likelihood of finding the right solution. This approach is articulated further by Spinczyk et al. (2019) who argued that increasing availability of structured content, providing ways for BVIs to independently access materials, and developing universal rules in the representation of structured information can contribute to a more consistent mathematical education for BVIs.

The third challenge, as outlined by Cryer et al. 2013, faced by BVI students in relation to accessing mathematics materials relates to teaching methods and the lack of training for teachers that specifically targets addressing the difficulties discussed above; for example, math lessons are taught in a “chalk and talk” (Cryer et al., 2013, p. 4) manner that can leave BVI students behind if they are not given proper preparation. As one student put it when asked about what they wished their teachers to know, “They need to do a better job of describing pictures” (Bell & Silverman, 2019, p. 6). What is more, there is an underlying misconception that students with BVI are not as cognitively capable as their sighted peers: “they should know that blind students are totally capable of learning math and science and excelling in those STEM fields” (Bell & Silverman, 2019, p. 6). Nevertheless, there is a will to learn present in the deeds and words of BVIs. Cryer et al. (2013) reported on several mathematicians who were blind who excelled in their field and postulated that their blindness might even offer an advantage because they are using their imaginations for space and number rather than relying solely on two-dimensional representations of them. For example, a study conducted in Norway (Klingenberg, 2012) had BVI students explore geometric concepts in three dimensions. The researcher found that body movements complemented manual exploration of geometric concepts because spatial awareness can be a link to future abstract thinking. BVIs seem to know what they are capable of: “I can do anything a sighted person can do” (Bell & Silverman, 2019, p. 6). How can this ability and willingness of BVIs to show growth in mathematics education be realized?

Given that NCTM (2000) defined mathematical representations as “processes and products that are observable externally as well as those that occur internally in the minds of people doing mathematics” (p. 67), it seems all the more important to provide BVIs with a mixed-representational approach when presenting mathematical materials (Fitzpatrick et al., 2018); there is not a “one size fits all” way of presenting technical information. BVIs use tactile and acoustic inputs to perceive the world around them, which means they experience it in a sequential way at first (Rottmann et al., 2020). This is why the adaptation of materials is necessary (Ahlberg & Csocsán, 1999); these materials act as the bridge from a sequential viewpoint to an understanding that is more well-rounded in nature. More research is needed that considers the effects of performance on math tasks when BVIs have access to a multisensory approach, and on how to teach BVIs to gather information from multiple sources efficiently (Beal & Rosenblum, 2018).

2.4. The creation of multi-sensory tools

Systematic differentiated instruction (Ketterlin-Geller et al., 2015) combined with inclusive assessments (Rottmann et al., 2020) could shed some light on best practices for the creation of tools that will allow BVIs to independently demonstrate growth in their mathematical content knowledge. Teachers must know the ongoing needs of their students and how to adapt to those needs in a timely manner (Ketterlin-Geller et al., 2015) to prevent BVIs from falling behind. The best time to start tackling this propensity to lag in mathematics is in middle school (Ketterlin-Geller et al., 2015) because success in algebra (normally taken in one's freshman year of high school) is a predictor for future college and career success. In particular, the topics taught in middle school that are most predictive of excellence in algebra include: expressions and equations (Klute et al., 2020), and properties of the natural number set and how these properties operate in the rational and integer number sets (Ketterlin-Geller et al., 2015). To determine the level of math knowledge that BVIs have acquired, an inclusive assessment should be used (Rottmann et al., 2020) that meets every test taker at their zone of proximal development and uses a growth point framework. The authors suggest implementing an already existing diagnostic tool because the tasks can stay the same, but the materials can be adapted for BVIs' use. It is important that the adaptation and creation of new tools does not outpace what goes on in the classroom (Scalise et al., 2018); the demands of the materials in any diagnostic task should match the needs of the student (Rottmann et al., 2020).

3. Conclusion

Overall, this shows that there truly is something to the old adage, "where there is a will, there is a way." Given proper academic supports within the school setting (Giesen et al., 2012), and access to multi- representational materials (Scalise et al., 2018) that engage multiple senses (Crollen & Collignon, 2020), many of the access barriers discussed above can be greatly reduced.

Given an inclusive assessment showing growth over time (Rottmann et al., 2020) with opportunities for BVIs to engage independently (Beal & Rosenblum, 2018), BVIs can demonstrate their mathematical knowledge to those around them. Perhaps the most important aspect of this research to keep in mind is that, with these supports outlined above, BVIs can perceive their own growth in mathematics and gain some much-needed confidence in their ability to succeed in technical fields of education.

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