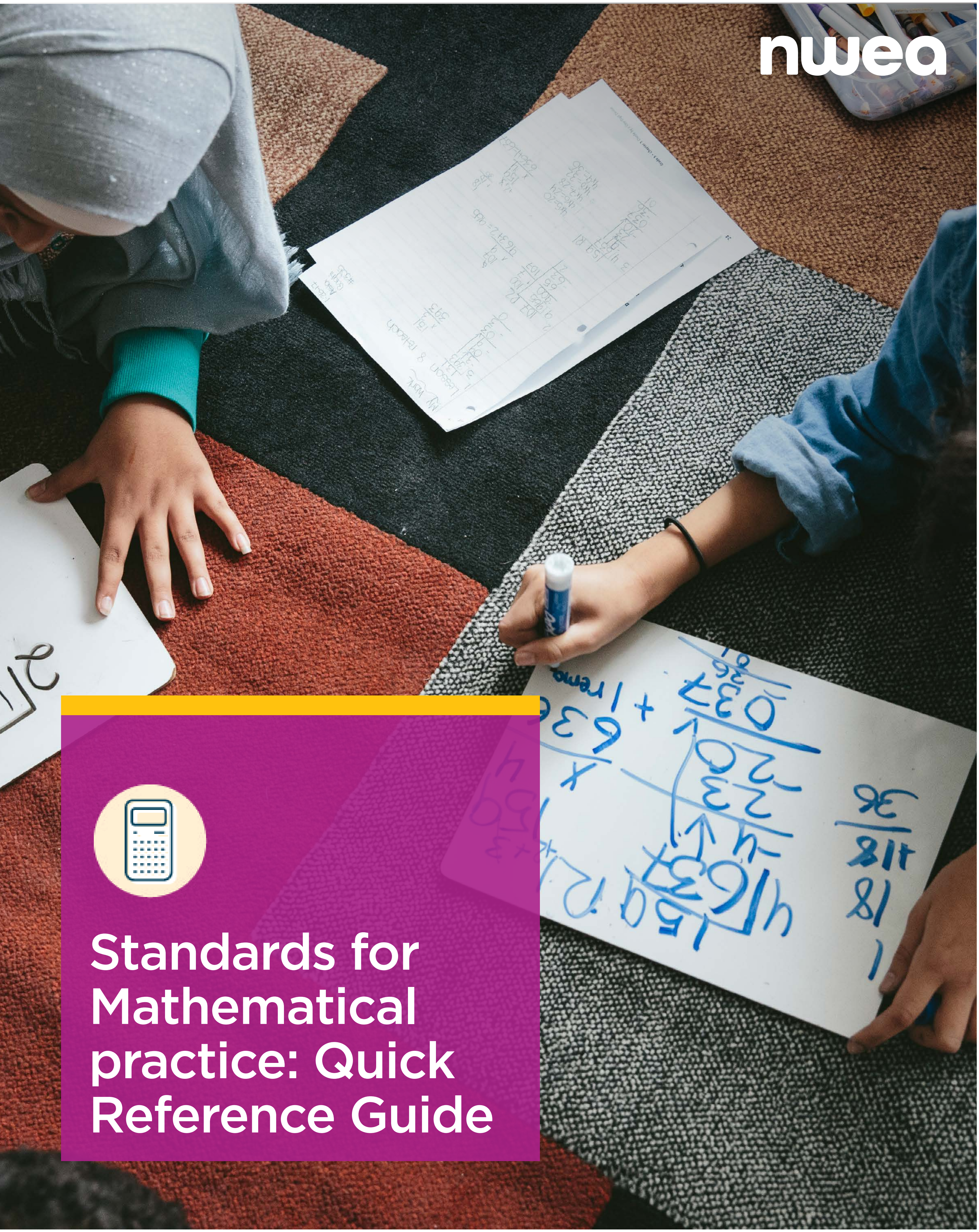

Standards for Mathematical practice: Quick Reference Guide

Introduction

The Standards for Mathematical Practice (SMP) support students' development of the mathematical skills and knowledge necessary to build a robust and connected understanding of mathematics. However, the standards are complex and challenging to interpret and implement (Bleiler et al., 2015; Olson et al., 2014). As a result, NWEA created quick guides to support teachers' implementation of mathematical practices in their classrooms.

What are the Standards for Mathematical Practice?

The authors of the Common Core State Standards [CCSS] (2010) compiled the SMP using the National Council of Teachers of Mathematics' (2000) five process standards and the National Research Council's (2001) standards of mathematical proficiency, resulting in a list of eight SMP.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

While the CCSS document lists the standards individually, Fuson (2023) suggests pairing the mathematical practices using four categories: math sense-making, math structure, math drawings, and math explaining (see Table 1). This allows teachers to provide more opportunities for students to engage with mathematical practices.

Table 1

Paired Standards for Mathematical Practice (Fuson, 2023)

Mathematical Practices			
Math Sense-Making	**Math Structure**	**Math Drawings**	**Math Explaining**
Make sense and use appropriate precision.	See structure and generalize.	Model and use tools.	Reason, explain, and question.
MP 1: Make sense of problems and persevere in solving them. MP 6: Attend to precision.	MP 7: Look for and make use of structure. MP 8: Look for and express regularity in repeated reasoning.	MP 4: Model with mathematics. MP 5: Use appropriate tools.	MP 2: Reason abstractly and quantitatively. MP 3: Construct viable arguments and critique the reasoning of others.
Teachers continually assist students to do math sense-making about math structure using math drawings to support math explaining.			

What is included in Standards for Mathematical Practice: Fractions?

The SMP description describes multiple skills students should master for each standard. This resource lists the targeted skills for each SMP and provides an example of what students might say or think related to an individual targeted skill before, during, and after solving a fraction task.

We believe that students can use rigorous academic math language. As a result, our potential student responses provide exemplary answers that use more formal language. We recognize not all students might use the same terminology.

This resource includes an easy-to-read chart of the targeted skills across all the SMP. In addition, eight quick guides, one for each mathematical practice, provide a list of targeted skills and an example of what students might say or think in response to each target skill through the lens of a fractions task.

How do I use this resource?

You can use this resource in many ways.

1. Print the Standards for Mathematical Practice: Target Skills chart.
2. Review by individual SMP Quick Guides.
3. Use the SMP Instructional Planning Tool to plan a lesson in your classroom.

Standards for Mathematical Practice: Targeted Skills

Math Sense-Making		Math Structure		Math Drawings		Math Explaining	
MP 1: Make sense of problems and persevere in solving them.	MP 6: Attend to precision.	MP 7: Look for and make use of structure.	MP 8: Look for and express regularity in repeated reasoning.	MP 4: Model with mathematics.	MP 5: Use appropriate tools.	MP 2: Reason abstractly and quantitatively.	MP 3: Construct viable arguments and critique the reasoning of others.
Explain the meaning of a problem. *Look for entry points to solve a problem.* *Analyze givens, constraints, relationships, and goals.* *Make conjectures about the form and meaning of the solution.* *Plan a solution pathway.* *Consider similar problems.* *Use simpler forms of the original problem to gain insight into its solution.* *Monitor and evaluate progress.* *Explain correspondences between equations, verbal descriptions, tables, and graphs.* *Search for regularity or trends.* *Draw diagrams of important features and relationships.* *Use concrete objects or pictures to conceptualize and solve a problem.* *Graph data.*	*Communicate precisely to others.* *Use clear definitions in discussion with others and their reasoning.* *State the meaning of the symbols chosen.* *Specify units of measure* *Label axes to clarify the correspondence with quantities in a problem.* *Calculate accurately and efficiently.* *Express numerical answers with a degree of precision appropriate for the problem context* *Provide formulated explanations to classmates.*	*Look for a pattern or structure.* *Identify a pattern or structure.* *Use the pattern or structure to solve the problem.* *Explain how the pattern or structure was used to solve the problem.*	*Look both for general methods and for shortcuts.* *Use both general methods and shortcuts to solve the problem.* *Notice if calculations are repeated.*	*Identify important quantities in a practical situation.* *Represent the relationships between quantities using models.* *Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.* *Make assumptions and approximations to simplify a complicated situation.* *Analyze those relationships mathematically to draw conclusions.* *Interpret mathematical results in the context of the situation.* *Reflect on whether the results make sense and improve the model if it has not served its purpose.*	*Consider the available tools when solving a mathematical problem.* *Select mathematical tools that would be useful in solving the problem.* *Determine mathematical tools that would not be useful in solving the problem.* *Identify relevant external mathematical resources.* *Use mathematics tools to pose or solve problems.* *Use technological tools to explore and deepen their understanding of concepts.*	*Make sense of quantities and their relationships in problem situations.* *Decontextualize a problem to focus on the quantities.* *Contextualize a problem to consider a real-life situation.* *Represent the problem symbolically.* *Manipulate the representing symbols as if they have a life of their own.* *Know and flexibly use different properties of operations and objects.* *Create a coherent representation of the problem.* *Attend to the meaning of quantities.* *Consider the units involved.*	*Understand and use stated assumptions, definitions, and previously established results in constructing arguments.* *Make conjectures.* *Build a logical progression of statements to explore the truth of conjectures.* *Analyze situations by breaking them into cases.* *Recognize and use counterexamples.* *Reason inductively about data, making plausible arguments that take into account the context from which the data arose.* *Construct arguments using concrete referents such as objects, drawings, diagrams, and actions.* *Justify their conclusions.* *Communicate them to others.* *Respond to the arguments of others.* *Listen/read the arguments of others.* *Compare the effectiveness of two plausible arguments.* *Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in an argument—explain what it is.*

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* National Governors Association Center for Best Practices and Council of Chief State School Officers.

Bleiler, S. K., Baxter, W. A., Stephens, D. C., & Barlow, A. T. (2015). Constructing meaning: Standards for mathematical practice. *Teaching Children Mathematics*, *21*(6), 336–344. https://www.jstor.org/stable/10.5951/teacchilmath.21.6.0336

Fuson, K. C. (2023, April 12). *Building math talk for sense-making in the classroom* [edWebinars]. https://home.edweb.net/webinar/math20230412/

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Olson, T. A., Olson, M., & Capen, S. (2014). The Common Core standards for mathematical practice: Teachers' initial perceptions and implementation considerations. *Journal of Mathematics Education Leadership*, *15*(2), 11–20.

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MP 1: Make sense of problems and persevere in solving them.

Read the entire mathematical practice standard here.

Targeted Skills

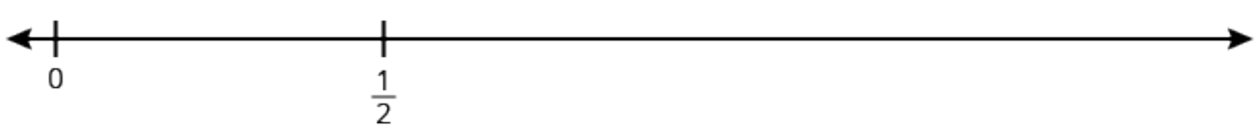
MP 1: Make sense of problems and persevere in solving them.	
Before solving, students will be able to:	• Explain the meaning of a problem. • Look for entry points to solve a problem. • Analyze givens, constraints, relationships, and goals. • Make conjectures about the form and meaning of the solution. • Plan a solution pathway. • Consider similar problems.
During solving, students will be able to:	• Use simpler forms of the original problem to gain insight into its solution. • Monitor and evaluate progress. • Explain correspondences between equations, verbal descriptions, tables, and graphs. • Search for regularity or trends. • Draw diagrams of important features and relationships. • Use concrete objects or pictures to conceptualize and solve a problem. • Graph data.
After solving, students will be able to:	• Check problem answers using a different method. • Ask themselves, "Does this make sense?". • Understand the approaches of others. • Identify correspondences between different approaches.

Grade 3 Fraction Task

Where is $\frac{4}{3}$ on the number line?

Aligned Content Standard

3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.



What do students need to make sense of before solving?

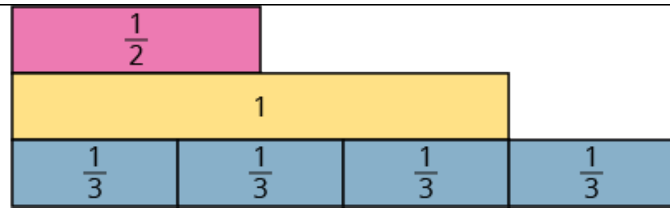
Before solving, students need to make sense of the problem and the relationships between quantities. They also need to plan for solving, which might include considering problems they previously encountered. For example, students might say or think:

Skill(s)	Potential Student Response
• Explain the meaning of a problem.	This problem is asking me to represent $\frac{4}{3}$ on a number line, where 0 and $\frac{1}{2}$ are already determined.
• Analyze givens, constraints, relationships, and goals.	I know that $\frac{4}{3}$ is 4 one-thirds. Since 3 one-thirds is the same as 1, I know $\frac{4}{3}$ is greater than 1.
• Look for entry points to solve a problem.	I see the $\frac{1}{2}$ on the number line. How can I use that to mark where 1 is? If $\frac{4}{3}$ is more than 1, do I need to mark where 2 is? How can I divide the number line into thirds?
• Consider similar problems.	I have used a number line to represent whole numbers. I have used an open number line to model adding fractions, like $\frac{1}{3}$ and $\frac{2}{3}$.
• Make conjectures about the form and meaning of the solution.	My solution will be a fraction on a number line because the problem asks me to find the location of the fraction $\frac{4}{3}$ on the provided number line.
• Plan a solution pathway.	To solve this problem, I want to divide the number line into equal parts, and I can use a ruler to determine the distance between $\frac{1}{2}$ and 1. Then, I can use this to determine the location for $1\frac{1}{2}$.

What do students need to make sense of during solving?

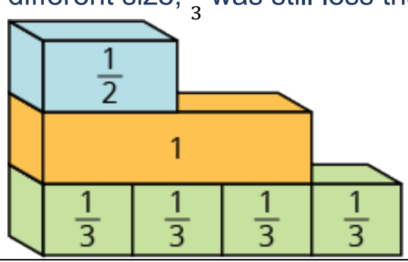
During solving, students can solve simpler problems or build different representations (e.g., concrete or pictorial) to work towards a problem solution. At the same time, they need to monitor their progress and evaluate if their work seems reasonable. For example, students might say or think:

Skill(s)	Potential Student Response
• Use simpler forms of the original problem to gain insight into its solution.	I will solve an easier problem first. I will locate $\frac{1}{3}$ on the number line. This will help me understand the relationship between $\frac{1}{3}$ and $\frac{1}{2}$. I notice that $\frac{1}{3}$ is less than $\frac{1}{2}$.
• Draw diagrams of important features and relationships.	I can use fraction tiles to represent the numbers in the problem, $\frac{1}{2}$, 1, and $\frac{4}{3}$.

• Use concrete objects or pictures to conceptualize and solve a problem.	
• Monitor and evaluate progress. • Search for regularity or trends. • Explain correspondences between equations, verbal descriptions, tables, and graphs.	My representation shows that $\frac{4}{3}$ is equal to 4 copies of $\frac{1}{3}$, which is 1 and $\frac{1}{3}$.
• Graph data	*This fraction task does not provide data for students to graph.*

What do students need to make sense of after solving?

After solving, students need to ask themselves, "Does my answer make sense?" and be able to compare their solution pathway with that of other students. For example, students might say or think:

Skill(s)	Potential Student Response
• Ask themselves, "Does this make sense?".	My answer makes sense because I placed $\frac{4}{3}$ between 1 and $1\frac{1}{2}$. Based on my representation, I know $\frac{4}{3}$ is greater than 1 and less than $1\frac{1}{2}$.
• Check problem answers using a different method.	I know if I divide the same whole into 2 equal parts that those parts will be bigger than if I divide the same whole into 3 equal parts. So, 3 one-thirds will be less than 3 one-halves.
• Understand the approaches of others. • Identify correspondences between different approaches.	I noticed that another student used Cuisenaire rods to represent the relationship between quantities. Because the whole was different, the length for $\frac{4}{3}$ was a different size for the concrete representation. Even though the whole was a different size, $\frac{4}{3}$ was still less than $1\frac{1}{2}$. 

MP 2: Reason abstractly and quantitatively.

Read the entire standard of mathematical practice here.

Targeted Skills

Before solving students will be able to:	• Make sense of quantities and their relationships in problem situations. • Decontextualize a problem to focus on the quantities. • Contextualize a problem to consider a real-life situation.
During solving students will be able to:	Decontextualize • Represent the problem symbolically. • Manipulate the representing symbols in ways that may or may not relate to their referents in the context of the problem. • Know and flexibly use different properties of operations and objects. Contextualize • Create a coherent representation of the problem. • Attend to the meaning of quantities. • Consider the units involved.
After solving students will be able to:	Determine if their answer makes sense based on the context of the problem.

Grade 4 Fraction Task

Each student in class needs $\frac{3}{8}$ cups of flour to make modeling clay. If there are 15 students in the class, how many cups of flour are needed?

Aligned Grade Content Standard

4.NF.B.4.c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

What do students need to reason with and quantify before solving?

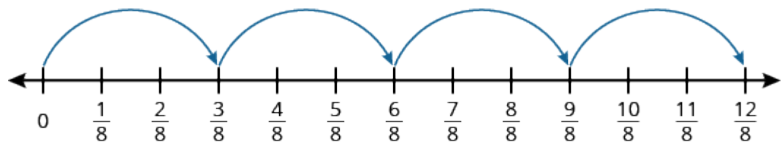
Before solving, students reason with the quantities and their relationship in the given problem. They might also need to contextualize a problem by thinking of a real-life situation or decontextualizing a problem to focus on the quantities. For example:

Skill(s)	Potential Student Response
• Make sense of quantities and their relationships in problem situations.	I notice the following numbers in the problem: $\frac{3}{8}$, and 15. I can understand the relationship between the numbers by thinking about what each number represents. The $\frac{3}{8}$ represents the number of cups of flour each student needs. The 15 represents the number of students in the class that need flour. Each student needs a little bit less than a half of a cup of flour.
• Decontextualize a problem to focus on the quantities.	I know that $\frac{3}{8}$ is a little bit less than $\frac{4}{8}$ which is the same as $\frac{1}{2}$ Since I need 15 groups of a little less than $\frac{1}{2}$, the answer will be less than half of 15 which is $7\frac{1}{2}$.

What do students need to reason with and quantify during solving?

During solving, students can use their recontextualized problem to help them reason with the abstract, decontextualized problem. The student might use the real-life scenario to draw a model that represents the relationship between quantities. For example:

Skill(s)	Potential Student Responses
• Represent the problem symbolically. • Manipulate the representing symbols as if they have a life of their own (without necessarily attending to their referents). • Know and flexibly use different properties of operations and objects.	I would write this problem as $\frac{3}{8}$ (15) is enough flour for all students. I could solve it as 45 x $\frac{1}{8}$, because I'm making 45 copies of $\frac{1}{8}$ which is $\frac{45}{8}$. Or I could solve it as $\frac{3}{8}$ x $\frac{15}{1}$ which is $\frac{(3 \, x \, 15)}{(8 \, x \, 1)}$ which is also $\frac{45}{8}$.
• Create a coherent representation of the problem at hand.	I will use a number line to represent multiplication. I'm measuring in $\frac{1}{8}$ cups, so my tick marks will each represent $\frac{1}{8}$. Since there are 15 students, I need 15 hops of 3-eighths to represent the total amount of cups of flour needed. This means that I need 45-eighths cups of flour, four hops shown here.

	
• Attend to the meaning of quantities.	Each $\frac{3}{8}$ hop represents the amount of flour needed by one student.
• Consider the units involved. • Contextualize a problem and consider the answer relative to real-life situation.	I can use whole cups as my unit of measurement. My answer is in eighths of a cup, but measuring eighths is less efficient than measuring whole cups. Since I need $\frac{45}{8}$ cups, I find out how many whole cups that is by dividing 45 by 8 to get $5\frac{5}{8}$ cups. This answer makes sense because our 15 students each need less than half a cup, I know I'll need less than half of 15 cups, and $5\frac{5}{8}$ cups is less than $7\frac{1}{2}$ cups of flour.

MP 3: Construct viable arguments and critique the reasoning of others.

Read the entire mathematical practice standard here.

Targeted Skills

MP 3: Construct viable arguments and critique the reasoning of others.	
Before the delivery of the argument, students will be able to:	• Understand and use stated assumptions, definitions, and previously established results in constructing arguments. • Make conjectures. • Build a logical progression of statements to explore the truth of conjectures. • Analyze situations by breaking them into cases. • Recognize and use counterexamples. • Reason inductively about data, making plausible arguments that take into account the context from which the data arose. • Construct arguments using concrete referents such as objects, drawings, diagrams, and actions.

During the delivery of the argument, students will be able to:	• Justify their conclusions.
	• Communicate them to others.
	• Listen/read the arguments of others and
	    ○ decide whether they make sense,
	    ○ ask useful questions to clarify or improve the argument.
	• Respond to the arguments of others

After the delivery of the argument, students will be able to:	• Compare the effectiveness of two plausible arguments. • Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

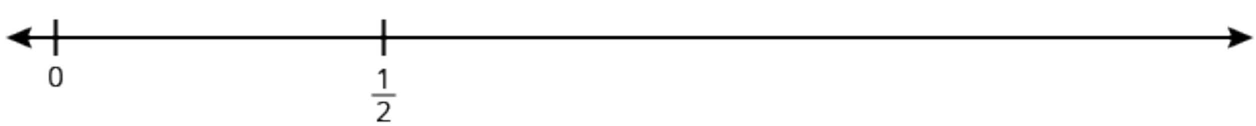
Note: In this context, an argument is not a back-and-forth debate between students, but an explanation that justifies and explains students' mathematical reasoning.

Grade 3 Fraction Task

Where is $\frac{4}{3}$ on the number line?

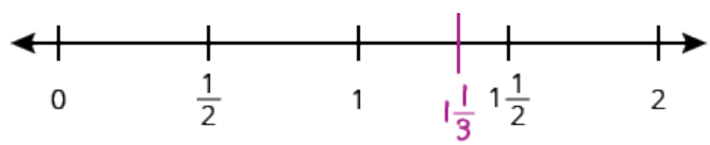
Aligned Grade Content Standard

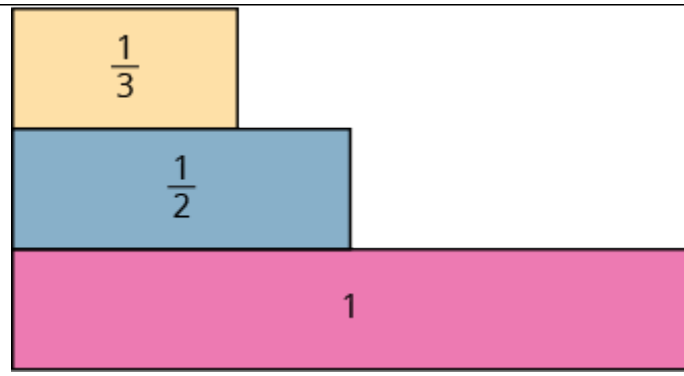
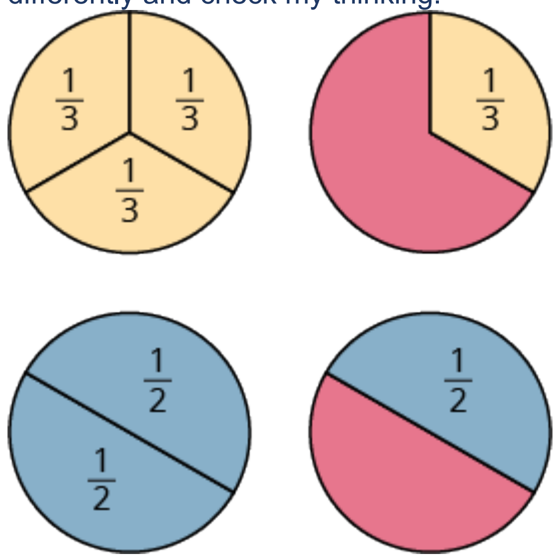
3.NF.A.2: Understand a fraction as a number on a number line; represent fractions on a number line diagram.



What do students need to do to construct viable arguments before they present their reasoning to the class?

Before students present their arguments, they need to construct a viable argument. An argument is not a back-and-forth debate between students, but an explanation that justifies and explains students' mathematical reasoning. Students will analyze the mathematical task, make connections to prior knowledge, solve the problem, determine a logical sequence to express their reasoning, and determine an appropriate representation to support their argument. For example:

Skill(s)	Potential Student Response
• Understand and use stated assumptions, definitions, and previously established results in constructing arguments.	This mathematical task is about fractions. I need to determine the location of $\frac{4}{3}$ on the number line. I know that $\frac{4}{3}$ is worth the same as 1 and $\frac{1}{3}$. The numerator tells me the number of equal parts. The denominator tells me the total number of equal parts in the whole.
• Make conjectures.	This is my answer: $\frac{4}{3}$ is located between 1 and $1\frac{1}{2}$ on the number line. Here is my visual representation to show my answer. 
• Build a logical progression of statements to explore the truth of conjectures. • Construct arguments using concrete referents such as objects, drawings, diagrams, and actions. • Recognize and use counterexamples.	Now I need to build an argument to prove that my answer is reasonable and justify my thinking for my classmates. $\frac{4}{3}$ is located on the number line between 1 and $1\frac{1}{2}$. I know this is correct because $\frac{4}{3}$ is greater than one whole. I know this because the numerator is larger than the denominator. The denominator tells me how many equal parts I need to make one whole. The numerator tells me how many equal parts I have: four equal parts. Three of those parts equals one whole with one part, or $\frac{1}{3}$, left over. This helps me know that $\frac{4}{3}$ is the same as $1\frac{1}{3}$. I know that $\frac{1}{3}$ is less than $\frac{1}{2}$, so $1\frac{1}{3}$ is before $1\frac{1}{2}$ on the number line. I used fraction tiles to prove this is right.

	 Finally, I used fraction circles to model my answer differently and check my thinking. 
• Analyze situations by breaking them into cases	The problem does not include different cases for students to analyze.
• Reason inductively about data, making plausible arguments that take into account the context from which the data arose.	The problem does not include data for students to reason with.

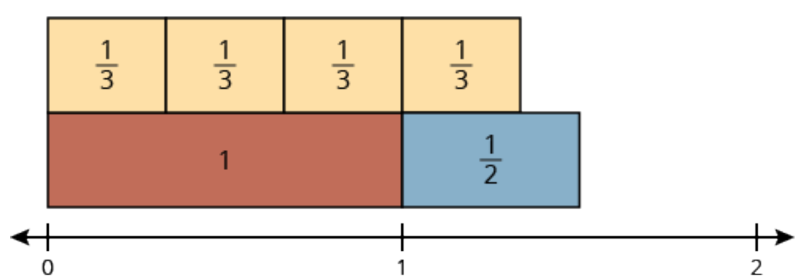
What do students need to do to present viable arguments and critique the reasoning of others while they share their reasoning?

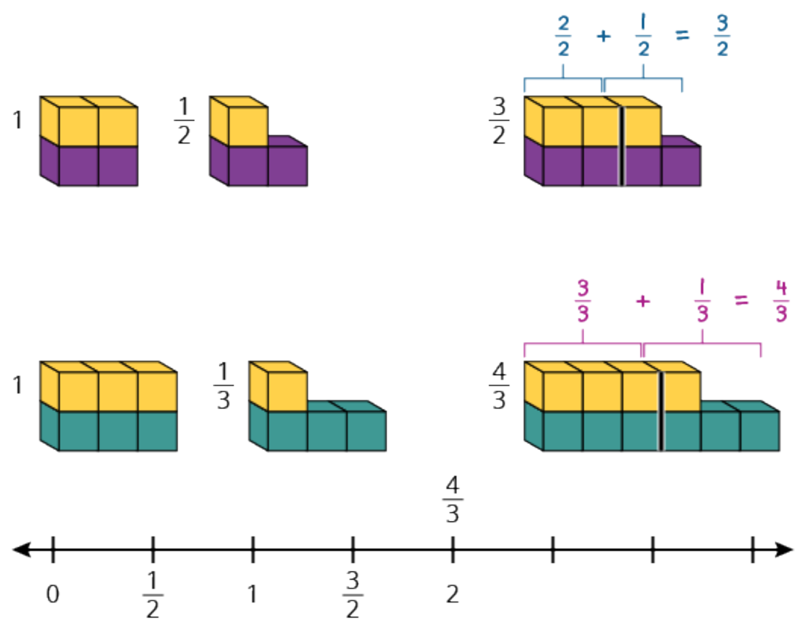
During the class discussion when students present their constructed arguments, they need to critique the reasoning of others and compare alternative solutions with their own. For example:

Skill(s)	Potential Student Responses
• Justify their conclusions. • Communicate them to others.	When I am presenting, I will read my answer and show each representation to help others understand how I got to my solution and why I think my solution is reasonable.
• Respond to the arguments of others.	If I am confused, I am going to ask clarifying questions like: How did you know that $\frac{4}{3}$ is the same as $1\frac{1}{3}$? Could you explain your thinking step by step? I am not sure I agree with your location on the number line. Can you show me a representation that supports your answer?
• Listen/read the arguments of others.	When others are presenting, I am going to listen to other solution pathways. I am looking at how others solved the problem to see if they used the same strategy as I did or a different one.

What do students need to do to critique the reasoning of others after everyone has shared their reasoning?

After the class discussion, it is helpful to give students time to critique and compare the reasoning of others with their own. For example:

Skill(s)	Potential Student Responses
• Compare the effectiveness of two plausible arguments.	This is another correct, alternative solution.  I noticed that this solution used fraction tiles and a number line. They were able to find the precise location of $\frac{4}{3}$ on a number line and I estimated the location of $\frac{4}{3}$ on a number line. Their solution is more precise. I think my strategy for solving was better. They did not use different representations to show that they knew $\frac{1}{3}$ is less

	than $\frac{1}{2}$. I think my second representation helped me better understand the meaning of $\frac{4}{3}$ and confirm that my answer was reasonable.
• Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.	I noticed an incorrect, alternative solution.  This answer is different from mine. I noticed that this person used Cuisenaire rods to show $1\frac{1}{2}$ and $\frac{4}{3}$. Based on their representation, they think the fraction $\frac{4}{3}$ is worth the same as 2. I do not agree. I think $\frac{6}{3}$ would be worth the same as 2. I found where they went wrong. They are not comparing fractions with the same whole. They need to use the same whole to compare fractions. I would help them use Cuisenaire rods by showing them how to iterate until they have a same-sized whole. Then they could use this strategy to compare $1\frac{1}{2}$ and $\frac{4}{3}$.

MP 4: Model with mathematics.

Read the entire mathematical practice standard here.

Targeted Skills

MP 4: Model with mathematics.	
Before solving, students will be able to:	• Identify important quantities in a practical situation. • Represent the relationships between quantities using models.
During solving, students will be able to:	• Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. • Make assumptions and approximations to simplify a complicated situation. • Analyze those relationships mathematically to draw conclusions.
After solving, students will be able to:	• Interpret mathematical results in the context of the situation. • Reflect on whether the results make sense and improve the model if it has not served its purpose.

Note: In this context, a model is a way to represent mathematical thinking. For example, students might use drawings, manipulatives, or digital tools to demonstrate how they solve a mathematical problem.

Grade 4 Fraction Task

Solve the following word problem.

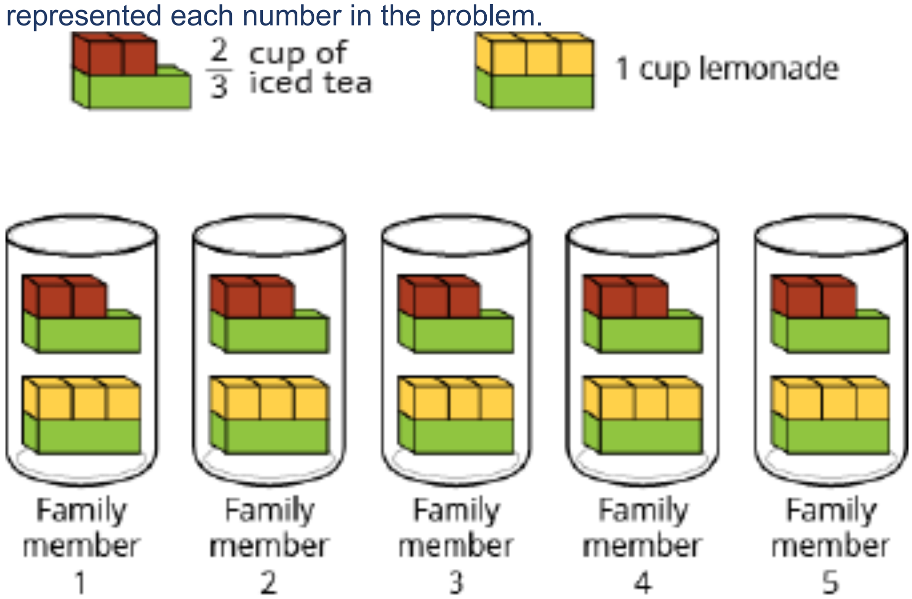
Aligned Grade Content Standard

4.NF.B.4.c Students will be able to solve word problems involving multiplication of a fraction by a whole number

Rafa makes himself a drink where he mixes $\frac{2}{3}$ cup of iced tea with 1 cup of lemonade. One afternoon, he makes the drink for all 5 members of his family. What is the total number of cups of the drink that he will make for his family?

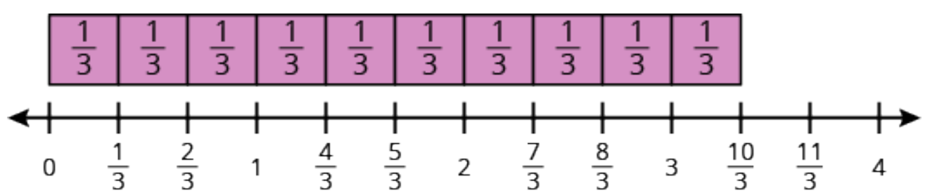
What do students need to do to model with mathematics before solving?

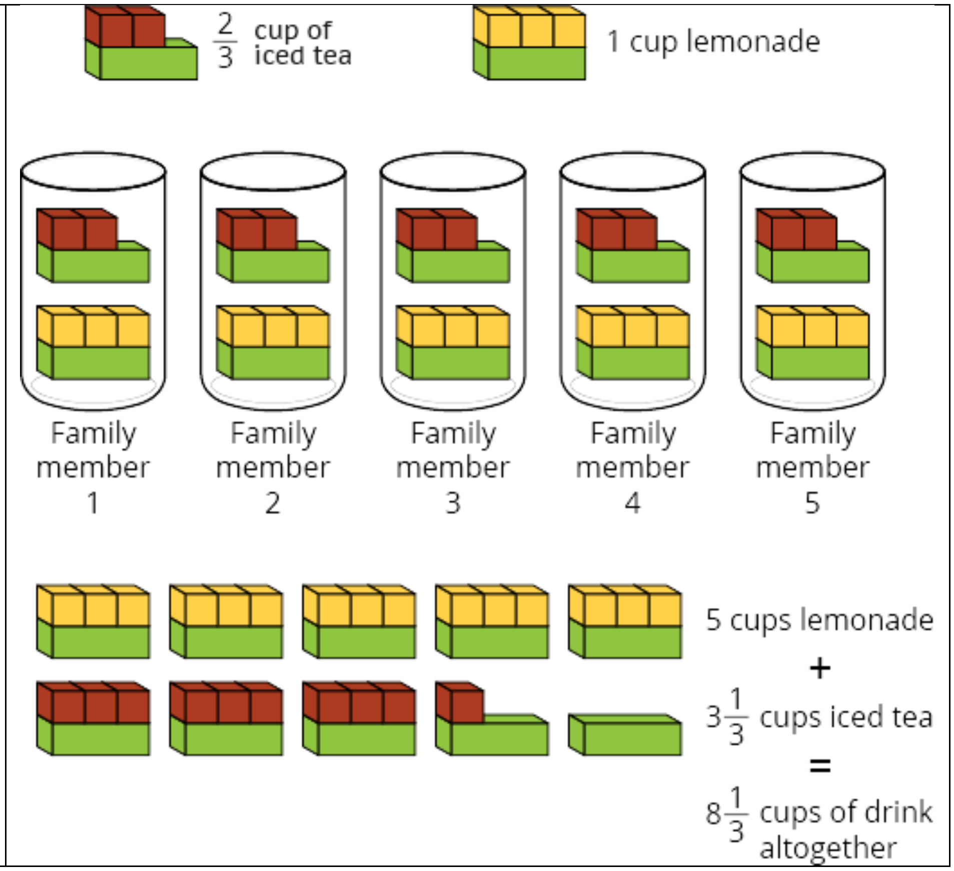
Before solving, students can represent the relationships between quantities using models. They need to identify the quantities in the problem and select appropriate models to represent the quantities. For example:

Skill(s)	Potential Student Response
• Identify important quantities in a practical situation.	I know that Rafa has 5 members in his family, so he will need to make five glasses of the drink. I know that each cup will have $\frac{2}{3}$ of a cup of iced tea and 1 cup of lemonade.
• Represent the relationships between quantities using models.	I am going to draw five empty cups to represent the total number of cups of the drink he will make. Each cup will have $\frac{2}{3}$ cup of iced tea and 1 cup of lemonade. I am going to use Cuisenaire rods to represent $\frac{2}{3}$ cup of iced tea and 1 cup of lemonade in each empty cup. I will use green as my whole. 1 cup of lemonade is the same as $\frac{3}{3}$. I am going to label my model to make sure I understand how I represented each number in the problem. 

What do students need to do to model mathematics during solving?

During solving, students can use models to reason with the mathematics in the problem and find a solution. Students can manipulate their models to find the solution to the problem. For example:

Skill(s)	Potential Student Response
• Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.	The question asks me to find the total number of cups of the drink Rafa will make for his family. I can use my model to help find the total number of cups he will make. First, I am going to find out the total number of cups in each family member's glass. There are $\frac{2}{3}$ cup of iced tea and 1 cup of lemonade. 1 cup is the same as $\frac{3}{3}$ cup. Each cup has $\frac{5}{3}$ cups of the drink or $1\frac{2}{3}$ cups of the drink.
• Make assumptions and approximations to simplify a complicated situation.	I can reorganize my model to find the total number of cups of the drink. I will count the total number of whole cups and the remaining cups left. I will reorganize the cups of lemonade. There are 5 cups of lemonade. I will reorganize the cups of iced tea. I can count by thirds, "$\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \frac{7}{3}, \frac{8}{3}, \frac{9}{3}, \frac{10}{3}$." I know $\frac{10}{3}$ is the same as $3\frac{1}{3}$ because I can move the unit fraction $\frac{1}{3}$ to make a whole cup. 
• Analyze those relationships mathematically to draw conclusions.	I can add together 5 cups of lemonade and $3\frac{1}{3}$ cups of iced tea by adding the whole numbers 5 + 3 = 8 and the fraction $\frac{1}{3}$. The total number of cups Rafa makes is $8\frac{1}{3}$ cups of the drink.


$\frac{2}{3}$ cup of iced tea
1 cup lemonade
Family member 1
Family member 2
Family member 3
Family member 4
Family member 5
5 cups lemonade
+
$3\frac{1}{3}$ cups iced tea
=
$8\frac{1}{3}$ cups of drink altogether

What do students need to do to model with mathematics after solving?

After solving, students can use models to explain their thinking, evaluate the reasonableness of their responses, and compare different solution pathways. For example:

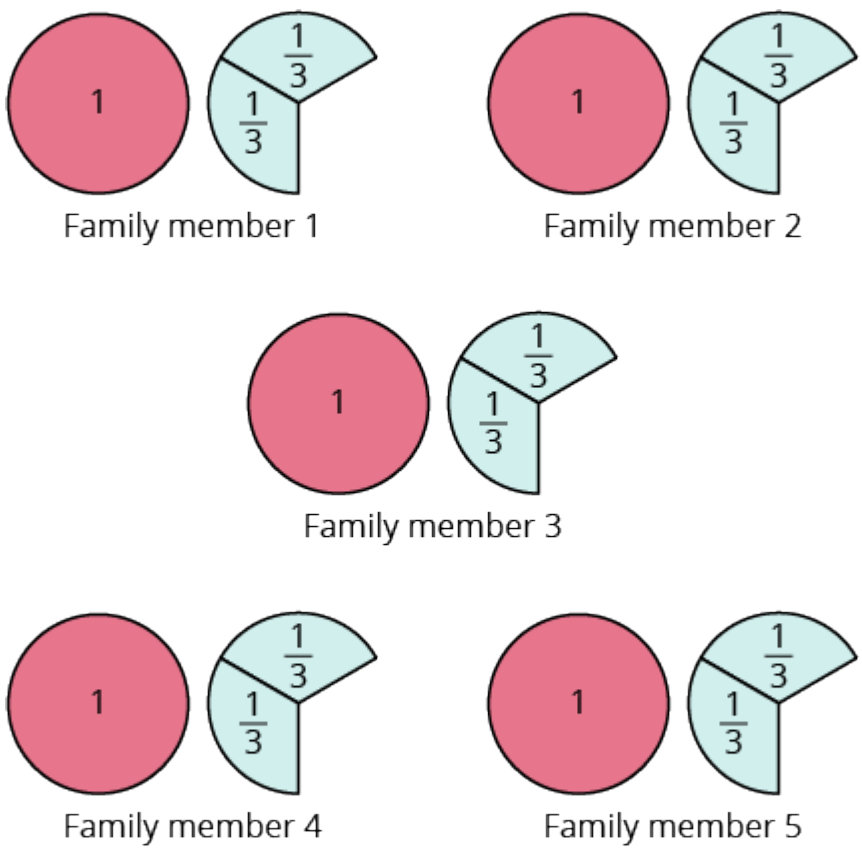
<table>
<tr><th>Skill(s)</th><th>Potential Student Response</th></tr>
<tr><td>

- Interpret mathematical results in the context of the situation

</td><td>

I can interpret my answer by examining my model. Rafa made $8\frac{1}{3}$ cups of the drink altogether for his family.

I can interpret my classmate's answers by examining their model. They found the answer using fraction circles. Instead of adding the total number of cups of lemonade and then the total number of cups of iced tea, they multiplied $1\frac{2}{3}$ cups of the drink by 5 glasses for each family member.



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<tr><td>

- Reflect on whether the results make sense and improve the model if it has not served its purpose.

</td><td>

I can check if my answer is reasonable by working backward and filling up each cup using my Cuisenaire rods. If I have the right amount in each cup at the end, I will know my answer makes sense.

</td></tr>
</table>

MP 5: Use appropriate tools strategically.

Read the entire mathematical practice standard here.

Targeted Skills

MP 5: Use appropriate tools strategically.	
Before solving, students will be able to:	• Consider the available tools when solving a mathematical problem. ○ For example: pencil and paper, concrete models, a ruler, a protractor, a calculator. • Select mathematical tools that would be useful in solving the problem. • Determine mathematical tools that would not be useful in solving the problem. • Identify relevant external mathematical resources.
During solving, students will be able to:	• Use mathematics tools to pose or solve problems. • Use technological tools to explore and deepen their understanding of concepts.
After solving, students will be able to:	This MP does not recommend specific skills for students to engage with after solving.

Grade 5 Fraction Task

Solve the following word problem.

Aligned Content Standard

5.NF.B.5.b Interpret multiplication as scaling (resizing), by: Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{nxa}{nxb}$ to the effect of multiplying $\frac{a}{b}$ by 1.

Ana has a piece of ribbon. Ben has $\frac{5}{3}$ as much ribbon as Ana. Sam has $\frac{3}{4}$ as much ribbon as Ana. Who has the longest piece of ribbon? Who has the shortest piece? How do you know?

What tools do students need to use strategically before solving?

Before solving, it is important that students determine which tools might be helpful when solving the problem; for example, students might say or think:

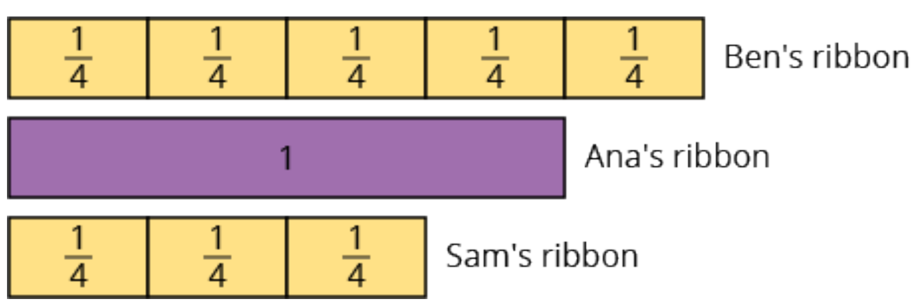
Skill(s)	Potential Student Response
• Consider the available tools when solving a mathematical problem.	My classroom has many math tools that might help me. This problem is about the length of a ribbon, so I might use a tool that shows a linear model like Cuisenaire rods, fraction tiles, or rulers.
• Select mathematical tools that would be useful in solving the problem.	The ribbon lengths provided are fractions. Cuisenaire rods and fraction tiles might be a more precise tool than a ruler. I will use fraction tiles because I feel most comfortable using this math tool. I can use Cuisenaire rods to check my answer for reasonableness.
• Determine mathematical tools that would not be useful in solving the problem.	I do not think it would be helpful to use base ten blocks or two-color counters because they help you build a model of a set, not a model to represent length.
• Identify relevant external mathematical resources.	If I get stuck on this problem, I can look at my notes for examples of similar problems, ask a classmate to explain their thinking, or ask my teacher for help.

What tools do students need to use strategically during solving?

When solving, students must determine what is mathematically important in the problem and what tool will help model or demonstrate how they are thinking about the answer to the problem.

<table>
<tr><th>Skill(s)</th><th>Potential Student Response</th></tr>
<tr><td>• Use mathematics tools to pose or solve problems.</td><td>
I can use fraction tiles to solve this problem. I built the following models to help me solve the problem. I tried different lengths for Ana’s length of ribbon. When I changed the length of Ana’s ribbon, it changed the lengths of Ben’s and Sam’s ribbons. The fraction tiles helped me see the different ways this problem might be answered.

Anna’s ribbon is 1 yard.



Ana’s ribbon is $\frac{2}{3}$ of a yard.
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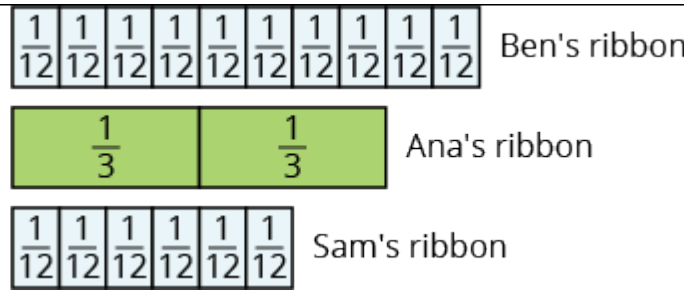
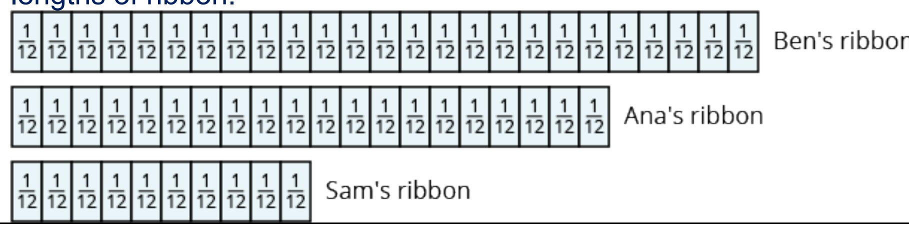


Ana’s ribbon is $\frac{5}{3}$, which is the same as $\frac{20}{12}$. Twelfths was the least common denominator, and it allowed me to compare the three lengths of ribbon.



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<td>

- Use technological tools to explore and deepen their understanding of concepts.

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<td>I can use virtual manipulatives to explore different ways to solve this problem. I might use virtual Cuisenaire rods and try different lengths for the whole.</td>
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MP 6: Attend to precision.

Read the entire mathematical practice standard here.

Targeted Skills

MP 6: Attend to precision.	
Before solving, students will be able to:	This MP does not recommend specific skills for students to engage in before solving.
During solving, students will be able to:	• Communicate precisely to others. • Use clear definitions in discussion with others and their reasoning. • State the meaning of the symbols chosen. o Use the equal sign consistently and appropriately. • Specify units of measure. • Label axes to clarify the correspondence with quantities in a problem. • Calculate accurately and efficiently.
After solving, students will be able to:	• Express numerical answers with a degree of precision appropriate for the problem context. • Provide formulated explanations to classmates.

Grade 5 Fraction Task

Solve the following word problem.

Aligned Content Standard

5.NF.B.5.b Interpret multiplication as scaling (resizing), by: Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{nxa}{nxb}$

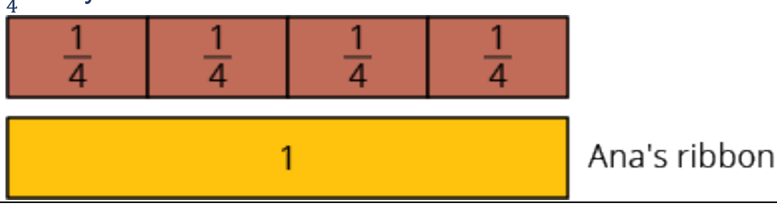
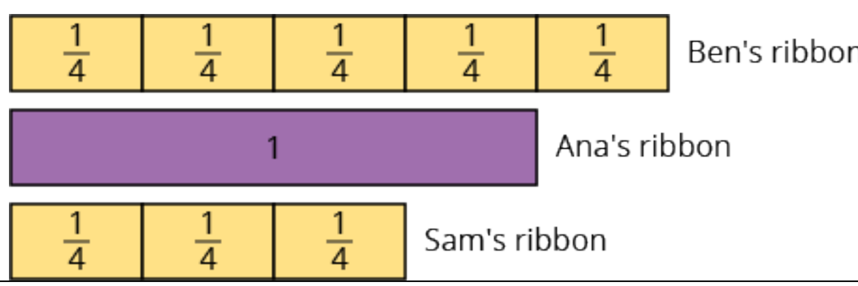
Ana has a piece of ribbon. Ben has $\frac{5}{4}$ as much ribbon as Ana. Sam has $\frac{3}{4}$ as much ribbon as Ana. Who has the longest piece of ribbon? Who has the shortest piece? How do you know?

How do students attend to precision during solving?

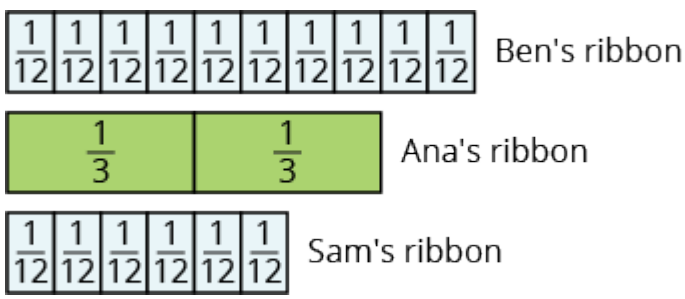
During solving, students need to use precision when communicating and making sense of the problem. Some students may make sense of the math using their invented strategies, but as more formal understanding develops, the use of precise math terminology is important.

Additionally, students might attend to precision by trying multiple solution strategies to confirm their initial response. The following example illustrates how a student might work through three different solution strategies to confirm their initial answer is precise. During this process, they will practice the targeted skills multiple times. For example, students might say or think:

Solution Strategy 1:

Skill(s)	Potential Student Responses
• Communicate precisely to others. • Specify units of measure. • Label axes to clarify the correspondence with quantities in a problem. • State the meaning of the symbols chosen.	I will select a length for Ana's ribbon to help me compare Ben's and Sam's ribbons. For this problem, Ana's ribbon will be 1 yard or $\frac{4}{4}$ of a yard. I will draw and label my representation to understand this problem better. I will use fraction tiles to represent Ana's ribbon length of 1 yard or $\frac{4}{4}$ of a yard. 
• Calculate accurately and efficiently.	Next, I will compare Ben's ribbon and Sam's ribbon with Ana's ribbon. I notice that $\frac{5}{4}$ x 1 = $\frac{5}{4}$ and $\frac{3}{4}$ x 1 = $\frac{3}{4}$. If Ana's ribbon is 1 yard, then Ben's ribbon is the longest and Sam's ribbon is the shortest. 
• Use clear definitions in discussion with others and their reasoning.	Now I need to check that this is true if the length of Ana's ribbon changes. Now Ana's ribbon is going to be $\frac{2}{3}$ of a yard, or less than one whole.

Solution Strategy 2

• Communicate precisely to others. • Specify units of measure. • State the meaning of the symbols chosen.	I will use what I know about multiplication to model my thinking and find a solution. $\frac{5}{4}$ of $\frac{2}{3}$ is the same as $\frac{5}{4}$ x $\frac{2}{3}$. I can multiply the numerators and the denominators to solve, and the answer tells me that Ben's ribbon is $\frac{10}{12}$ of a yard, which is longer than Ana's ribbon. I know that $\frac{2}{3}$ is equivalent to $\frac{8}{12}$, so $\frac{10}{12}$ is longer than $\frac{8}{12}$. I can use symbols to show my thinking. Ben's ribbon is longer than Ana's ribbon. I can use a greater than symbol to compare the values: $\frac{10}{12} > \frac{2}{3}$. $\frac{3}{4}$ of $\frac{2}{3}$ is the same as $\frac{3}{4}$ x $\frac{2}{3}$. I can multiply the numerators and the denominators to solve, and the answer tells me that Sam's ribbon is $\frac{6}{12}$ of a yard. Sam's ribbon is the shortest. I can use symbols to show my thinking. Sam's ribbon is shorter than Ana's ribbon. I can use the less than symbol to compare the values: $\frac{6}{12} < \frac{2}{3}$. 
• Calculate accurately and efficiently.	I notice that when Ana's ribbon is less than 1 whole, her ribbon is the second longest. In both examples, Ben's ribbon is longer than Sam's ribbon.
• Use clear definitions in discussion with others and their reasoning.	To make my answer even more precise, I will try another length for Ana's ribbon that is longer than one. Now Ana's ribbon is 20 inches long, which is more than one whole.

Solution Strategy 3

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- Communicate precisely to others.
- Specify units of measure.
- State the meaning of the symbols chosen.
- Calculate accurately and efficiently.

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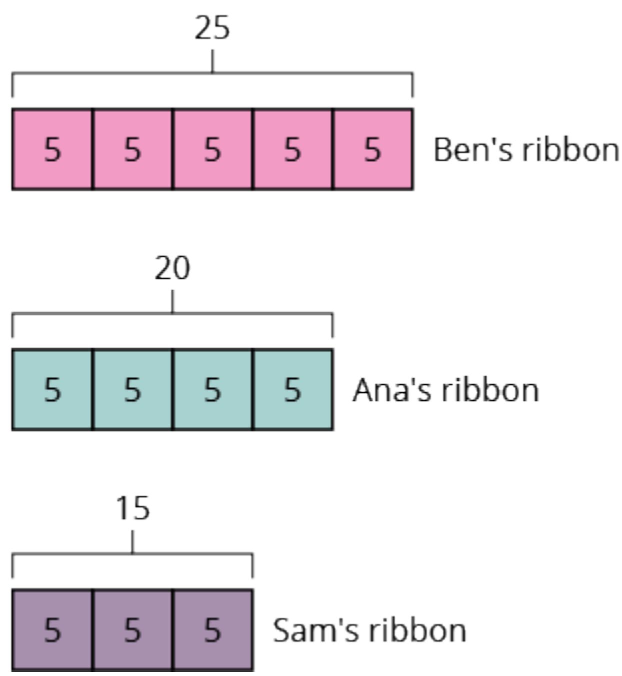
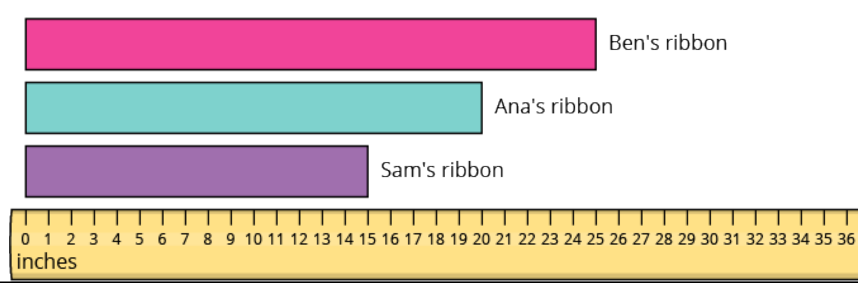
$\frac{5}{4}$ of 20 is the same as $\frac{5}{4}$ x $\frac{20}{1}$. I can multiply the numerators and the denominators to solve. The answer I got is $\frac{100}{4}$. $\frac{100}{4}$ simplifies to 25 inches. Ben's ribbon is 25 inches, which is longer than Ana's ribbon.

$\frac{3}{4}$ of 20 is the same as $\frac{3}{4}$ x $\frac{20}{1}$. I can multiply to find the answer and I get $\frac{60}{4}$. $\frac{60}{4}$ simplifies to 15 inches. Sam's ribbon is shorter than Ana's ribbon.

I can use symbols to compare the quantities.
25 inches > 20 inches
15 inches < 20 inches
25 inches > 15 inches



OR



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How do students attend to precision after solving?

After solving, students need to provide an answer that gives an appropriate level of detail and be able to clearly explain their reasoning to classmates. For example, students might say or think:

Skill(s)	Potential Student Responses
• Express numerical answers with a degree of precision appropriate for the problem context.	When Ana's ribbon is 1 yard, Ben's ribbon is $1\frac{1}{4}$ yard and Sam's ribbon is $\frac{3}{4}$ yard. When Ana's ribbon is $\frac{2}{3}$ of a yard, Ben's ribbon is $\frac{10}{12}$ of a yard and Sam's ribbon is $\frac{6}{12}$ of a yard. When Ana's ribbon is $\frac{5}{3}$ of a yard, Ben's ribbon is $\frac{25}{12}$ of a yard and Sam's ribbon is $\frac{15}{12}$ of a yard. In all three scenarios, Ben's ribbon is the longest and Sam's ribbon is the shortest.
• Provide formulated explanations to classmates.	Based on my examples, it does not matter if Ana's ribbon is less than, equal to, or more than one whole. Ben's ribbon will always be the longest and Sam's ribbon will always be the shortest.

MP 7: Look for and make use of structure.

Read the entire mathematical practice standard here.

Targeted Skills

MP 7: Look for and make use of structure.	
Before solving, students will be able to:	• Look for a pattern or structure. Examples: ○ Communitive Property - noticing that three and seven more is the same amount as seven and three more. ○ Classifying Objects - sorting a collection of shapes according to identified features. ○ Distributive Property - seeing 7 × 8 equals 7 × 5 + 7 × 3. ○ Decomposing Numbers - seeing the 14 as 2 × 7 and the 9 as 2 + 7.
During solving, students will be able to:	• Use the pattern or structure to solve the problem.
After solving, students will be able to:	• Explain how the pattern or structure was used to solve the problem.

Grade 4 Fraction Task

Solve the following word problem.

Aligned Content Standard

4.NF.B.4.c: Solve word problems involving multiplication of a fraction by a whole number.

Rafa makes himself a drink where he mixes $\frac{2}{3}$ of a cup of iced tea with 1 cup of lemonade. One afternoon, he makes the drink for all 5 members of his family. What is the total number of cups of the drink that he will make for his family?

How do students look for and make use of structure before solving?

Before solving, students can look for and make use of structure by examining the relationship between the quantities in the word problem. For example, students might say or think:

Skill(s)	Potential Student Responses
• Look for a pattern or structure.	The problem is asking me to find the total number of cups of the drink for five glasses, or groups. I know that multiplication is equal to groups of the same amount. I am going to multiply 5 by the total amount of the drink in each glass.

	I know there is 1 cup of lemonade and $\frac{2}{3}$ cup of iced tea. That makes $1\frac{2}{3}$ cups of the drink in each glass. So, I need to solve to find out how many total cups are in five glasses of $1\frac{2}{3}$ cup of the drink, or $5 \times 1\frac{2}{3}$.

How do students look for and make use of structure during solving?

During solving, students can use the pattern or structure to solve the problem. For example, students might say or think:

Skill(s)	Potential Student Responses
• Use the pattern or structure to solve the problem.	I drew a picture of each glass with $1\frac{2}{3}$ cups of the drink in each glass. Now I notice that they all have the same amount. This reminds me of repeated addition. I will add $1\frac{2}{3}$ five times. There are five glasses with $1\frac{2}{3}$ cups of the drink in each glass. I can add $1\frac{2}{3}$ cups together five times, or $1\frac{2}{3} + 1\frac{2}{3} + 1\frac{2}{3} + 1\frac{2}{3} + 1\frac{2}{3}$.

How do students look for and make use of structure after solving?

After solving, it is important for students to compare the different structures and patterns students discerned before and during solving. This will illustrate the different solution pathways students can take and the underlying mathematical concepts that are related. For example, students might say or think:

Skill(s)	Potential Student Responses
• Explain how the pattern or structure was used to solve the problem.	I noticed that one student used multiplication and solved $5 \times 1\frac{2}{3}$. I noticed another student used repeated addition and solved $1\frac{2}{3} + 1\frac{2}{3} + 1\frac{2}{3} + 1\frac{2}{3} + 1\frac{2}{3}$. I thought about it differently, and I used the distributive property to help me. I decomposed $1\frac{2}{3}$ into 1 cup of lemonade and $\frac{2}{3}$ cup of iced tea. Then I multiplied each amount with 5 glasses. I solved $(5 \times 1) + (5 \times \frac{2}{3})$.

MP 8: Look for and express regularity in repeated reasoning.

Read the entire standard of mathematical practice here.

Targeted Skills

MP 8: Look for and express regularity in repeated reasoning.	
Before solving, students will be able to:	• Look both for general methods and shortcuts.
During solving, students will be able to:	• Use both general methods and shortcuts to solve the problem. • Notice if calculations are repeated. • For example, notice when dividing 25 by 11 that they are repeating the same calculations over and over again.
After solving, students will be able to:	This MP does not recommend specific skills to execute after solving.

Note: We define shortcuts as formulas where students follow specific steps and general methods as the other solution pathways students might take (before learning or mastering the formula) to solve a problem.

Aligned Content Standard

4.NF.B.4.c: Solve word problems involving multiplication of a fraction by a whole number.

Grade 4 Fraction Task

Solve the following word problem.

Rafa makes himself a drink where he mixes $1\frac{2}{3}$ of a cup of iced tea with 1 cup of lemonade. One afternoon, he makes the drink for all 5 members of his family. What is the total number of cups of the drink that he will make for his family?

How do students look for and express regularity in repeated reasoning before solving?

Before solving, students can look for general methods or shortcuts for solving the problem. Students might remember a formula that they have used on a similar problem. Alternatively, they might invent a solution pathway based on the problem context using general mathematical methods. It is helpful for students to try to solve a problem in more than one way. This allows them to make connections between the different methods. For example, students might say or think:

Skill(s)	Potential Student Responses
• Look both for general methods and shortcuts.	I can solve this problem in different ways. First, I could solve this problem by adding $1\frac{2}{3}$ cup of iced tea five times and adding 1 cup of lemonade five times. Next, I would add the sum from each problem together to find the total cups of the drink. I would solve: $1\frac{2}{3} + 1\frac{2}{3} + 1\frac{2}{3} + 1\frac{2}{3} + 1\frac{2}{3}$ = x cups of iced tea 1 + 1 + 1 + 1 + 1 = y cups of lemonade x + y = total cups of the drink Another way I can solve this problem is by using an equation: Number of glasses * (Iced Tea + Lemonade) = Total cups of the drink. If I used the equation, I would solve 5 x ($1\frac{2}{3}$ + 1) = Total cups of the drink.

How do students look for and express regularity in repeated reasoning during solving?

During solving, students can use the formulas or solution strategies they determine before solving. They can also notice if they repeat calculations to enhance their efficiency in solving current or future problems. For example, students might say or think:

Skill(s)	Potential Student Responses
• Use both general methods and shortcuts to solve the problem.	I can solve: $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$ = x cups of iced tea 1 + 1 + 1 + 1 + 1 = y cups of lemonade x + y = total cups of the drink First, I added the numerators together. The denominator stays the same. $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{10}{3}$ cups of iced tea Next, I added the whole numbers together by counting by 1. 1 + 1 + 1 + 1 + 1 = 5 cups of lemonade

	Now I am going to add the total number of cups of iced tea and lemonade together. $\frac{10}{3}$ + 5 First, I need to change $\frac{10}{3}$ to a mixed number. $\frac{10}{3} = 3\frac{1}{3}$ Last, I will add the whole numbers together (3 + 5) and the fraction ($\frac{1}{3}$) to get a total of $8\frac{1}{3}$. $3\frac{1}{3} + 5$ Rafa will make $8\frac{1}{3}$ cups of the drink for his family. Using repeated addition helped my relation that I can solve this faster using multiplication. I can solve 5 x ($\frac{2}{3}$ + 1) . 5 x ($\frac{2}{3}$ + 1) First, I added $\frac{2}{3}$ and 1 together. 5 x ($1\frac{2}{3}$) Next, I changed $1\frac{2}{3}$ to an improper fraction. 5 x ($\frac{5}{3}$) Then, I changed 5 to the fraction $\frac{5}{1}$. $\frac{5}{1}$ x $\frac{5}{3}$ Now, I multiplied the numerators, and I multiplied the denominators. $\frac{5x5}{1x3}$ When I simplify $\frac{25}{3}$, I get $8\frac{1}{3}$. $\frac{25}{3} = 8\frac{1}{3}$ Rafa will make $8\frac{1}{3}$ cups of the drink for his family.
• Notice if calculations are repeated.	In the first problem, I did not repeat calculations. It was the most efficient way to solve the problem. If I understand why the formula works and use the rule to take fewer steps, then it is called a shortcut. If I don't understand why the formula works, then it's called a trick. In the second problem, I added $\frac{2}{3}$ and 1 repeatedly, a total of five times, when I used repeated addition. This was one way I could solve the problem and it helped me understand what each number meant. Using this strategy helped me understand why the formula worked. Instead of repeated addition, I could have solved: (5 x $\frac{2}{3}$) + (5 x 1).

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