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Formative Conversation Starters

Student understanding is more about growing than it is about getting. As educators, we might speak of some students "getting it" and other students not. This conclusion, though, is not fair to students. A student who may not seem to "get it" does understand *something*, even if that *something* may not yet have grown into a robust web of thinking. To reach students' full knowledge, we can use guided conversations: Formative Conversation Starters.

Purpose: The purpose of Formative Conversation Starters is to help teachers reveal student understanding about key ideas in mathematics and to identify their students' ways of thinking.

Audience: The intended audience is teachers, who will use these questioning strategies with students.

Application: Teachers may wish to use these conversation starters in one-on-one conferences with students or in small groups.

Formative Conversation Starters approach student knowledge by presenting a single standards-based assessment item and leveraging the item to elicit conversation through clustered questioning. The goal of this activity is not to tell students what to think, but to help teachers better uncover how students are currently thinking about mathematical concepts. The conversations provide opportunities for students to communicate how they are thinking about mathematics.

Mathematical Ideas (BINSS – Big Ideas to Nurture Standards Sense-making)

As you read through these items, you will notice that we draw attention to a few specific mathematical ideas. These ideas correspond to important ways of thinking that all students should develop and continue to refine. They include:

- **Operations:** Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

- **Place Value:** Knowledge of place value is essential, and students should develop ways of thinking about place value that enable them to see the relationships between places. For example, they can think of a value in one place as 10 times that same value in the place to the right (*or a bundle of 10*), and they can carry that thinking between any places in any direction. They should be able to use that understanding effortlessly to compose and decompose quantities and to connect place-value understanding to operations.

- **Comparisons:** Comparisons can be either additive or multiplicative, with context guiding which is most appropriate. A multiplicative comparison is relative, describing one quantity in terms of another (e.g., 6 meters is <u>3 times as large</u> as 2 meters). Additive comparisons are absolute; the comparison is based on some other quantity (e.g., 6 meters is <u>4 meters more</u> than 2 meters). Students should have ways of thinking that help them determine which comparison to use or how an existing comparison is additive or multiplicative.

- **Measurement:** Geometric measurement is ultimately understood as the result of a multiplicative comparison between common attributes of two measurable quantities, and the result describes how many copies of a are contained in b. Equivalently, measurement addresses a times-as-large comparison such as "a is n times as large as b." Students thinking about measurement should have a clear understanding of which attribute is being measured and the comparison of two objects with that attribute, where one object's attribute is measured in terms of the other.

- **Fractions:** A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

- **Formulas:** Mastery of formulas (and procedures) is not the goal. Formulas are not the mathematics; they should be seen as shortcuts to help accomplish something with the mathematics. Students should have ways of thinking about the formulas that enable them to make sense of the quantities and to determine why quantities are connected with the indicated operations. Students should have mental and mathematical ways to reinvent useful formulas (e.g., $A=\pi r^2$ means three-and-a-bit copies of the square with area r^2).

- **Variables:** Students should have ways of thinking that enable them to distinguish between unknowns that vary and unknowns that represent some fixed value. For example, in the equation $6=3x+2$, x represents some unknown fixed value that makes the equation true. In $y=3x+2$, y and x vary with each other. In $y=mx+b$, y and x vary with each other, while m and b are typically nonvarying constants (parameters) within a problem.

- **Covarying Quantities:** Single quantities can vary, but students also need to consider situations where two quantities vary together. For example, in the equation $y=3x+2$, as the quantity x varies, the quantity y varies. It can be helpful to think about the relationship between covarying quantities in terms of how changes in x result in changes in y (e.g., as x increases by 1, y changes by . . .).

- **Proportional Relationships:** Proportional relationships require two covarying quantities. Those quantities must be measurable in some way, and the measures of those quantities scale in tandem. When one quantity changes by a scale factor, the other quantity also changes by the same scale factor. For example, doubling one quantity's value results in a doubling of the other quantity's value. Students should have ways of thinking that allow them to distinguish the two varying quantities in any proportional relationship and to explain how the quantities change by the same scale factor. It is important to note that proportional relationships are not synonymous with proportions. See this whitepaper for more information.

- **The Equal Sign:** The equal sign works in multiple ways in mathematics, even though the symbol does not change. Students should be able to think about the symbol as being relational (e.g., 2+4=5+1) and as being operational (e.g., the output of a computation), and should be able to determine which role the symbol is playing based on the situation. Keep an eye out for when students may put together equal signs. If this happens, prompting the student with questions about what the equals sign means may help. For example, suppose a student writes: 9 x 8 = 72 = 126 − 72 = 54 (*in this case, the student might be using the equal sign to say "next, I will..."*).

In the Formative Conversation Starters, you will notice that we call out mathematical ideas and the ways of thinking associated with them when it makes sense to do so. We also group discussion prompts to focus the conversations. Some groupings target core understandings that underpin the content. Other groupings elicit flexibility of thinking or extend beyond the assessment item being discussed. All of these methods are intended to provide opportunities for the teacher to listen to students and to reflect on how students might be thinking about mathematics in the standards-based assessment item.

The progressive question-and-answer strategy can be used to elicit evidence of students' ways of thinking about a topic or concept, with the purpose of guiding instruction.

How to Conduct a Formative Conversation

1. The questions for each item were developed to help teachers elicit information about students' ways of thinking about the content in the item and about mathematical ideas. These questions are suggestions, however, and not intended to be used as a script. The conversations teachers have will vary by student. While the questions for an item are laid out in a progression, teachers should vary the order to adapt to students' responses. Teachers should also keep in mind that students' responses may point to ways of thinking that are not addressed by the provided questions. In these cases, teachers should pursue those student understandings with their own line of questioning. There are several actions teachers can take to prepare for formative conversations:

   a. Become very familiar with the task and the questions ahead of time. This ensures that teachers can select the most appropriate next question based on how the students are responding.

   b. Provide students with tools to help them answer the questions. Depending on the task, these tools might include a manipulative, drawing paper, graph paper, or individual whiteboards and markers.

   c. Have a list of questions to help further probe what students are thinking. Some examples are:

      i. Can you tell me more about that?

      ii. You look like you're really thinking about this. What are you thinking?

      iii. Can you draw me a picture/write an equation?

      iv. How did you get that answer?

      v. Is there another way that you could find that answer?

   d. Make a plan to track what students say during the conversation: record the conversation, take notes, or have an observer take notes.

2. Teachers should ask questions without judgment. Student responses should not be labeled as right or wrong, and follow-up questions should be asked regardless of whether students give a correct response. Teachers should avoid commenting on students' responses other than to ask follow-up questions or to clarify what a student has said. Other students, however, should be encouraged to agree or disagree in a small group setting.

3. One of the most important parts of the formative conversation is what comes after the conversation: how will a teacher use the information about student thinking when planning instruction. Consider these suggestions for how to act on a formative conversation:

   a. Identify the different mathematical ideas addressed in the formative conversation. Where did students make connections between the ideas? Where do the connections need to be strengthened?

   b. Identify what students already understand in order to build instruction on that understanding.

   c. Identify areas where students can deepen what they already understand.

   d. Identify ways that students are comfortable with expressing mathematical ideas, and plan how to expand their capabilities. How were students most comfortable expressing or explaining what they understand? Were they more inclined to create a graphical representation of their thinking? Did they prefer to explain verbally? Did they use equations, or did they prefer to use graphs?

4. Some Conversation Starters include extension questions. These questions are provided as a way to elicit thinking beyond the grade-level of the BINSS.

3.1 Operations

This activity focuses on student thinking about operations, comparisons, and making sense of the unknown.

Operations: Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

ITEM ALIGNMENT

CCSS: 3.OA.8

This item focuses on solving real-world problems using operations. However, it also provides opportunities to talk about the meaning of operations, operations and quantities, and comparison.

THE CONVERSATION STARTER

Use the information to answer the question.

Last year, a squirrel collected 126 acorns to save for the winter. So far this year, it has collected acorns for 8 days. Each day, the squirrel has collected 9 acorns.

How many more acorns does the squirrel need to collect this year to equal the number collected last year? Enter the answer in the box.

[   ] acorns

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

What are you trying to solve in this problem?

- Can you explain what the question is asking?
- Is this a comparison question? Explain.
  - Can you describe what is being compared?

B. Problem Solving: Strategy

How do you want to approach this question?

Which numbers do you want to work with first? Why?

- How does this relationship help you answer the question?

C. Content: Operations (Multiplication)

Let's talk about multiplication in general. What does multiplication do for us?

- Can I multiply any two numbers?
- Can you think of a real-world context involving two quantities where multiplication does not make sense?

D. Content: Operations and Expressions (Representation)

The problem says 126 acorns were collected last year, and that 9 acorns have been collected each day for 8 days this year. Given these quantities, what could the following expressions represent?

- $126 - 9$
- $126 + 9$
- $126 \div 9$
- 126×9

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

What are you trying to solve in this problem?

- Can you explain what the question is asking?

  Rather than simply repeating the question, ask students to put the question in their own words to elicit understanding of the comparison made.

- Is this a comparison question? Explain.

  Yes. The number of acorns collected so far this year is being compared to the number of acorns collected last year.

  - Can you describe what is being compared?

    The number of acorns the squirrel got last year, or 126, compared to the number of acorns the squirrel has so far this year, which is 8 groups of 9, or 72.

B. Problem Solving: Strategy

How do you want to approach this question?

Listen to how students want to approach problems like this. Some may want to think quietly, and some may want to draw a picture. Based on the answer, follow up on student flexibility of other approaches.

Which numbers do you want to work with first? Why?

Even though they are not the first numbers mentioned in the problem, it makes sense to start by multiplying the 8 and 9 to find the number of acorns collected so far this year.

- How does this relationship help you answer the question?

  We need to know how many acorns have been collected so far before we can determine how many more acorns are left to collect to reach 126 acorns.

C. Content: Operations (Multiplication)

Let's talk about multiplication in general. What does multiplication do for us?

It can tell us how many things we have total when we have m ***groups of size*** n ***each. It can "copy" to compare something that is*** n ***times the size of something else.***

- Can I multiply any two numbers?

  Numbers? Yes. But sometimes in real life it might not make sense to do so.

- Can you think of a real-world context involving two quantities where multiplication does not make sense?

  For example, if there are 10 third graders and 10 fourth graders, it would not make sense to multiply 10 x 10. In the context of the problem, it would not make sense to multiply 126 acorns by 8 days.

D. Content: Operations and Expressions (Representation)

The problem says 126 acorns were collected last year, and that 9 acorns have been collected each day for 8 days this year. Given these quantities, what could the following expressions represent?

- $126 - 9$

  It could represent last year's total collection compared to one day of collected acorns this year.

- 126 + 9

  It could represent last year's total collection combined with the first day of this year's collection.

- 126 ÷ 9

  It could represent how many days it would take to equal last year's collection collecting at the same pace as this year.

- 126 × 9

  It would represent the number collected last year multiplied by the number collected each day this year, but it does not make sense to multiply these.

3.1 Shareables*

Use the information to answer the question.

Last year, a squirrel collected 126 acorns to save for the winter. So far this year, it has collected acorns for 8 days. Each day, the squirrel has collected 9 acorns.

How many more acorns does the squirrel need to collect this year to equal the number collected last year? Enter the answer in the box.

[ ] acorns

3.2 Operations

This activity focuses on student thinking about operations, comparisons, and making sense of the unknown.

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ITEM ALIGNMENT

CCSS 3.OA.8

This item focuses on operations. However, it also provides practice with problem solving, multiplication and division, remainders, and commutativity.

THE CONVERSATION STARTER

Use the information to answer the question.

Javier's photo album fits 9 pictures on each page. He filled 7 pages and has some pictures on page 8.

Which could be the total number of pictures that Javier has in the album?

- A. 50
- B. 58
- C. 66
- D. 74

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Orientation

Can you explain in your own words what the question is asking?

- What do you know?
- What do you need to know?
- Is there anything that is challenging about the problem situation?
    - What does it mean that there are "some pictures on page 8"?
- What is the first thing you want to do? Why?

B. Content: Operations (Multiplication, Division)

In general, what does multiplication do?

In general, what does division do?

- What are some real-life situations in which I may need to find 12÷3?
    - Would it tell me how many inches long each piece would be if I split a 12-inch length into three equal pieces?
    - Would it tell me how many 3-inch sections I can cut from a 12-inch length?
    - Would it tell me that 12 is 4 times as large as 3?
    - Which of these is closest to your example? Why?

C. Content: Operations (Division with Remainder)

How are these expressions alike? How are they different?

- What is a remainder?
- When you divide a number by 3, can you end up with a remainder of 4? Why or why not?
    - What should be the actual remainder in that case?
    - How many possible remainders are there when you divide by 5?
- **Extension:** Operations (Division with Remainder)

  We know that:

  13÷3=4 with a remainder of 1

  9÷2=4 with a remainder of 1

  Do both "4 with a remainder of 1" results represent the same value? Explain.

D. Content: Operations (Multiplication, Commutativity)

How are these expressions alike? How are they different?

- How could these expressions represent two different situations with pictures and pages?

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Orientation

Can you explain in your own words what the question is asking?

The question is about finding a possible number of pictures in the album given that there are 7 full pages each with 9 pictures and an 8th page that has some pictures but is not full.

- What do you know?

  You know that 9 pictures fit on one page. Javier has 7 pages, or 7 groups of 9 photos. Javier also has some pictures on page 8.

- What do you need to know?

  How many pictures are on page 8.

- Is there anything that is challenging about the problem situation?

  Finding how many pictures are on page 8.

  - What does it mean that there are "some pictures on page 8?"

    It means Javier fills each page completely with 9 pictures. Those photos filled up 7 full pages with some leftover, which are on page 8.

- What is the first thing you want to do? Why?

  A student may choose to multiply 9 x 7 first to find the pictures on the full pages.

B. Content: Operations (Multiplication, Division)

In general, what does multiplication do?

It can tell us how many things we have total when we have m ***groups of size*** n ***each. It can "copy" to compare something that is*** n ***times the size of something else.***

In general, what does division do?

Depending upon the context, a ÷ b can be used to determine how many groups of b ***are contained in*** a***. Or, a ÷ b can be used to determine how many are in each group if*** a ***is cut up into*** b ***equal groups. It can also compare two quantities to tell how many copies of one are in another—to determine how many "times as large as."***

- What are some real-life situations in which I may need to find 12÷3?

  Listen to student responses for how many groups, how many are in each group, or comparison thinking.

  - Would it tell me how many inches long each piece would be if I split a 12-inch length into three equal pieces?

    Yes: The group size is 4".

  - Would it tell me how many 3-inch sections I can cut from a 12-inch length?

    Yes: The number of groups (sections) is 4.

  - Would it tell me that 12 is 4 times as large as 3?

    Yes: It compares and tells "times as large as."

  - Which of these is closest to your example? Why?

    Listen for the ability to recognize the model of division the student employed.

C. Content: Operations (Division with Remainder)

How are these expressions alike? How are they different?

In both cases, we can think about cutting a number up into groups of three or into three equal groups. In both cases, there are 4 groups of 3 or there are 3 groups with 4 in each group. However, in 12 ÷ 3, there are no "leftovers," but with 13 ÷ 3, we are left with 1 leftover, which is called the remainder.

$$12 \div 3 \qquad 13 \div 3$$

- What is a remainder?

  Briefly, it's the leftover amount. Suppose we start with a quantity, a, ***and split it into groups of size*** b. ***The leftover amount that is not enough to create another group of size*** b ***is the remainder. Alternatively, suppose we want to split quantity*** c ***into*** d ***equal-sized groups. This is only possible with whole numbers when*** c ***is a multiple of*** d. ***If not, some amount will fall short for sharing across each group. That leftover amount is the remainder.***

- When you divide a number by 3 can you end up with a remainder of 4? Why or why not?

  Possible remainders when dividing by 3 are 0, 1, and 2. Since the remainder is 4, there is one more group of 3 (or group of size 3) that should have been included.

  - What should be the actual remainder in that case?

    1

  - How many possible remainders are there when you divide by 5?

    There are 5: 0, 1, 2, 3, 4.

- **Extension:** Operations (Division with Remainder)

  We know that:

  13 ÷ 3 = 4 with a remainder of 1

  9 ÷ 2 = 4 with a remainder of 1

  Do both "4 with a remainder of 1" results represent the same value? Explain.

  While both division problems have the same answer, 4 with a remainder of 1, what is represented is not the same. For example, in the first division problem, we can ask: How many 3s are there in 13? The answer is that there are 4 whole 3s and ⅓ of another 3. In the second problem we can ask: How many 2s are there in 9? We know that there are 4 whole 2s and ½ of another 2.

D. Content: Operations (Multiplication, Commutativity)

How are these expressions alike? How are they different?

From a calculation standpoint, they are both equal to 63. From a contextual standpoint, however, they might mean different things. In this situation, for example, if the first expression represents 9 pictures on each of 7 pages, then the second expression would mean 7 pictures on each of 9 pages. Those are two different things, but both have the same number of pictures total.

- How could these expressions represent two different situations with pictures and pages?

$$9 \times 7 \qquad 7 \times 9$$

  See above.

3.2 Shareables*

Use the information to answer the question.

Javier's photo album fits 9 pictures on each page. He filled 7 pages and has some pictures on page 8.

Which could be the total number of pictures that Javier has in the album?

- A. 50
- B. 58
- C. 66
- D. 74

C.

$12 \div 3 \qquad 13 \div 3$

D.

$9 \times 7 \qquad 7 \times 9$

3.3 Fractions

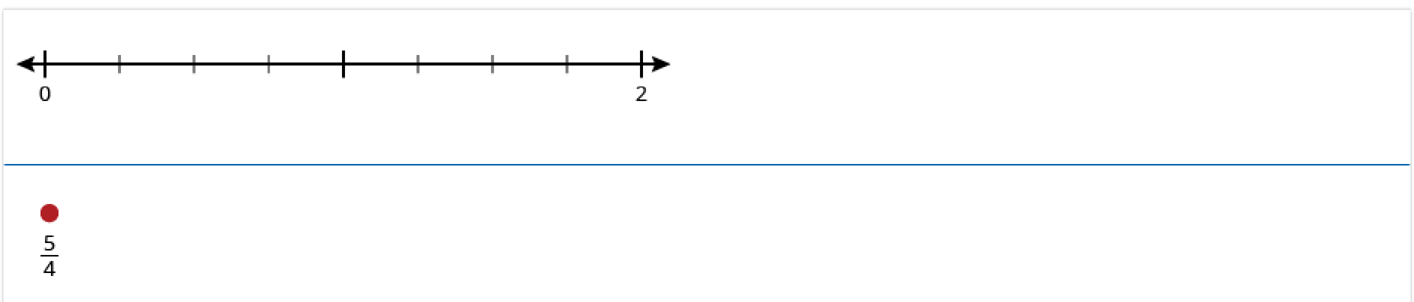
This activity focuses on fractions, connecting whole number representation of numbers on a number line to fraction representation, and understanding of the numerator and denominator.

Fractions: A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

ITEM ALIGNMENT	
CCSS 3.NF.A.2b	*This item focuses on placing a fraction on a number line. However, it also provides an opportunity to talk about the meaning of fractions, fractions as numbers on the number line, and comparison.*

THE CONVERSATION STARTER

Place the fraction at the correct location on the number line.



CONVERSATION PATHS (QUESTIONS ONLY)*

A. Content: Fraction (Meaning)

What is a fraction?

- Is a fraction two numbers or one number?
- What other words do we use with fractions? What do they mean?
- What do the a and b mean in the fraction a/b?

B. Content: Number Line (Meaning)

What do the tick marks represent on this number line?

- Is $5/3$ on the number line?
- Where would $5/3$ be on the number line?

C. Content: Fraction (Meaning)

What does $5/4$ mean?

- Can you use multiplication to write $5/4$ in another form?
- Can you use addition to write $5/4$ in another form?
- Is $5/4$ greater than, less than, or equal to $5/3$?
- How might you use the number line in the question to support your answer?

D. Content: Fraction (Comparison)

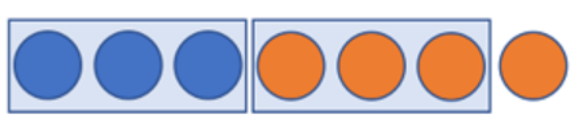
How much bigger is $4/4$ than $1/4$?

- How much bigger is $5/4$ than $1/4$?
- How many copies of $4/4$ are in $8/4$?

E. Content: Fraction (Representation)

How might you draw your own number line to show $7/3$?

- How could you use objects to show $7/3$?
- Tell me when you see $7/3$ in the image below.



CONVERSATION PATHS (ANNOTATED)*

A. Content: Fraction (Meaning)

What is a fraction?

A fraction is a number just as 3, 12, or 0 are numbers. It represents a point on the number line.

- Is a fraction two numbers or one number?

  A fraction looks like two numbers, but it is a single number.

- What other words do we use with fractions? What do they mean?

  Numerator and denominator. The denominator tells the number of partitions of the whole, and the numerator tells how many copies of that partition we have.

- What do the a and b mean in the fraction a/b?

  The quantity "b" tells us that a whole unit was cut up into "b" equal parts. Each of these parts is called "1/b." The quantity "a" tells us how many copies of "1/b" we are considering with "a/b."

B. Content: Number Line (Meaning)

What do the tick marks represent on this number line?

Each tick mark represents $\frac{1}{4}$.

- Is $\frac{5}{3}$ on the number line?

  Yes, but it isn't shown with a tick mark.

- Where would $\frac{5}{3}$ be on the number line?

  Between the tick marks for $\frac{6}{4}$ and $\frac{7}{4}$.

C. Content: Fraction (Meaning)

What does $\frac{5}{4}$ mean?

5 copies of $\frac{1}{4}$ where $\frac{1}{4}$ is the quantity obtained when a whole is cut into 4 equal pieces or segments.

- Can you use multiplication to write $\frac{5}{4}$ in another form?

  For example, $5 \times \frac{1}{4}$

- Can you use addition to write $\frac{5}{4}$ in another form?

  For example, $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

- Is $\frac{5}{4}$ greater than, less than, or equal to $\frac{5}{3}$?

  $\frac{5}{4}$ is 5 copies of $\frac{1}{4}$. $\frac{5}{3}$ is 5 copies of $\frac{1}{3}$. Since $\frac{1}{3}$ is larger than $\frac{1}{4}$, 5 copies of $\frac{1}{3}$, or $\frac{5}{3}$, is larger than 5 copies of $\frac{1}{4}$, or $\frac{5}{4}$.

- How might you use the number line in the question to support your answer?

  In each case, there are 5 copies of something. Since cutting a segment of length 1 into 4 equal segments produces smaller segments than cutting a segment of length 1 into 3 equal segments, the number line shows that $\frac{5}{3}$ is a greater length than $\frac{5}{4}$.

D. Content: Fraction (Comparison)

How much bigger is $\frac{4}{4}$ than $\frac{1}{4}$?

To further leverage the ambiguity around "bigger," the thinking can be additive or multiplicative. Do your students see the multiplicative approach? Can they articulate that $\frac{4}{4}$ is 4 times as large as $\frac{1}{4}$ (4 copies of $\frac{1}{4}$ compared to 1 copy of $\frac{1}{4}$)?

- How much bigger is $\frac{5}{4}$ than $\frac{1}{4}$?

  Can students articulate that $\frac{5}{4}$ is 5 times as large as $\frac{1}{4}$? Can they articulate how $\frac{5}{4}$ is $\frac{4}{4}$ larger than $\frac{1}{4}$?

- How many copies of $\frac{4}{4}$ are in $\frac{8}{4}$?

  There are 2 copies of $\frac{4}{4}$ in $\frac{8}{4}$. $\frac{4}{4}$ is 4 copies of $\frac{1}{4}$ and $\frac{8}{4}$ is 8 copies of $\frac{1}{4}$. Therefore, there are 2 copies of $\frac{4}{4}$ in $\frac{8}{4}$.

E. Content: Fractions (Representation)

How might you draw your own number line to show $\frac{7}{3}$?

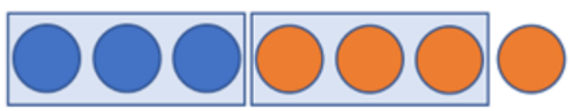
Start with a number line showing a segment that is at least one whole in length. For example, a segment that starts at zero and ends at 1. Cut this length into 3 equal parts. Copy this segment a total of 7 times.

- How could you use objects to show $\frac{7}{3}$?

  One way is to create a set of 3 objects to represent a whole. Each object is $\frac{1}{3}$ of the whole set. Seven of the objects represent $\frac{7}{3}$.

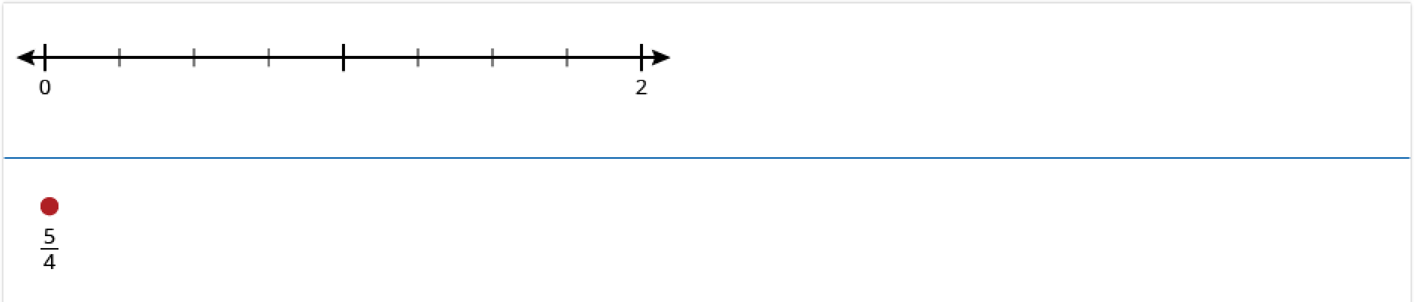
- Tell me when you see $\frac{7}{3}$ in the image below.

  Suppose that 3 circles make a whole. One circle is $\frac{1}{3}$. The 7 circles represent $\frac{7}{3}$.

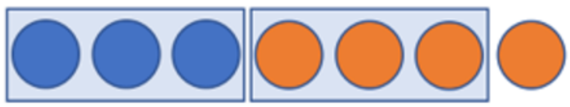


3.3 Shareables*

Place the fraction at the correct location on the number line.



E.



3.4 Fractions

This activity focuses on student thinking about fraction concepts, comparisons, and making sense of the numerator and denominator.

Fractions: A fraction is a single number. It is a number just as 1, or 100, or 37,549 are, and it has a location on the number line. Students should be able to think of a fraction as a number and treat it as such. The fraction a/b can be thought of as a copies of $1/b$, where $1/b$ is the length of a single part when the interval from 0 to 1 is partitioned into b parts. Two fractions are equivalent when they share a location on the number line.

ITEM ALIGNMENT

CCSS: 3.NF.A.3d

This item focuses on comparing fractions. However, it also provides an opportunity to talk about the meaning of fractions and the importance of comparison based on the same whole.

THE CONVERSATION STARTER

Move a symbol to each box to make true comparisons.

$\frac{5}{3} \square \frac{5}{4}$

$\frac{2}{8} \square \frac{2}{6}$

$=$ $<$ $>$

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Content: Fraction (Meaning)

- What is a fraction?
  - Is a fraction two numbers or one number?
  - What is the first thing you think of when you see $\frac{2}{8}$?
  - What other words do we use with fractions? What do they mean?
  - What do the a and b mean in the fraction a/b?

B. Content: Fraction (Comparison)

This problem uses "greater than" or "less than." What makes one fraction greater than another fraction?

- Which number is larger?

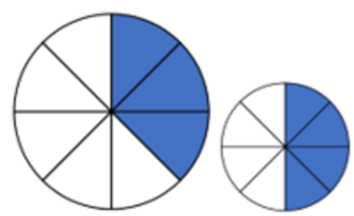
$$321 \qquad \mathbf{\Large 123}$$

- What makes two fractions equal?

C. Content: Fraction (Comparison, Importance of Whole)

Which is bigger: $\frac{3}{8}$ or $\frac{1}{2}$?

$$\frac{3}{8} \text{ or } \frac{1}{2}$$



D. Content: Fractions (Meaning, Comparison)

How are these two fractions alike? How are they different?

$$5/3 \qquad 5/4$$

- How could you model how they are different?

How are these two fractions alike? How are they different?

$$2/5 \qquad 4/5$$

- How could you model how they are different?
- Which is bigger: $\frac{2}{5}$ or $\frac{4}{5}$? Explain how you know.
- Which is larger: $\frac{3}{7}$ or $\frac{4}{9}$? Explain how you know.

CONVERSATION PATHS (ANNOTATED)*

A. Content: Fraction (Meaning)

- What is a fraction?

  A fraction is a number just as 3, 12, or 0 are numbers. It represents a point on the number line.

- Is a fraction two numbers or one number?

  A fraction looks like two numbers, but it is a single number.

- What is the first thing you think of when you see $2/8$?

  Listen to how students are thinking. Some students may describe the 2 and 8 separately, as in one is the numerator and one is the denominator. Other students may relate the 2 and 8, as in the fraction represents 2 one-eighth parts of a whole.

- What other words do we use with fractions? What do they mean?

  Numerator and denominator. The denominator tells the number of partitions of the whole, and the numerator tells how many copies of that partition we have.

- What do the a and b mean in the fraction a/b?

  b ***is the number of partitions of the whole and*** **a** ***is the number of those partitions.***

B. Content: Fraction (Comparison)

This problem uses "greater than" or "less than." What makes one fraction greater than another fraction?

One fraction is greater than another if it represents a point farther right on the number line.

- Which number is larger?

  321   **123**

  It depends upon the attribute you are talking about; we don't know yet. The number 123 is taller, but 321 would be farther right on the number line.

- What makes two fractions equal?

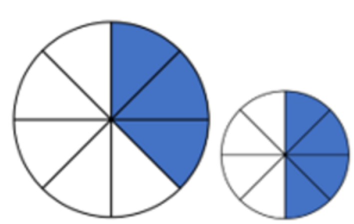
  Two fractions are equal if they can be placed in the same location on a number line. For example, ½ and 4/8 occupy the same location on a number line.

C. Content: Fraction (Comparison, Importance of Whole)

Which is bigger: 3/8 or ½?

These are tandem questions to elicit dissonance. First show the fractions, then show the circle diagrams. The use of "bigger" here is intentionally ambiguous to drive conversation. Do we mean bigger in value on the number line or physically bigger? If it means <u>bigger in value on a number line</u>, then students have to match the wholes.

$\frac{3}{8}$ or $\frac{1}{2}$



D. Content: Fractions (Meaning, Comparison)

How are these two fractions alike? How are they different?

$$5/3 \qquad 5/4$$

In both cases, there are 5 copies of something. However, in $\frac{5}{4}$, there are 5 copies of $\frac{1}{4}$, and in $\frac{5}{3}$ there are 5 copies of $\frac{1}{3}$.

- How could you model how they are different?

  We could start with the same size whole (rectangle, circle, etc.) and cut it into either 3 or 4 equal-size pieces.

How are these two fractions alike? How are they different?

$$2/5 \qquad 4/5$$

In both cases, we are talking about a whole that is divided into 5 partitions. However, in $\frac{2}{5}$ there are 2 copies of $\frac{1}{5}$, and in $\frac{4}{5}$ there are 4 copies of $\frac{1}{5}$.

How could you model how they are different?

You could show two rectangles each divided into 5 equal parts. In one rectangle, shade 2 parts to show $\frac{2}{5}$. In the other rectangle, shade 4 parts to show $\frac{4}{5}$. The number of shaded parts is not the same.

- Which is bigger: $\frac{2}{5}$ or $\frac{4}{5}$? Explain how you know.

  $\frac{4}{5}$ is bigger since 4 copies of $\frac{1}{5}$ is a larger value than 2 copies of that same $\frac{1}{5}$—where $\frac{1}{5}$ is obtained by cutting a whole unit into 5 equal p ieces.

- Which is larger: $\frac{3}{7}$ or $\frac{4}{9}$? Explain how you know.

  Listen for the strategy students use. One strategy may be to create equivalent fractions with common numerators: $\frac{3}{7} = \frac{12}{28}$ and $\frac{4}{9} = \frac{12}{27}$. Since $\frac{1}{27}$ is larger than $\frac{1}{28}$, 12 copies of $\frac{1}{27}$ is larger than 12 copies of $\frac{1}{28}$. So, $\frac{4}{9}$ is larger than $\frac{3}{7}$. Another strategy is to use common denominators: $\frac{3}{7} = \frac{27}{63}$ and $\frac{4}{9} = \frac{28}{63}$. Since 28 copies of $\frac{1}{63}$ is larger than 27 copies of $\frac{1}{63}$, we conclude that $\frac{4}{9}$ is larger than $\frac{3}{7}$.

3.4 Shareables*

Move a symbol to each box to make true comparisons.

$\frac{5}{3}$ ☐ $\frac{5}{4}$

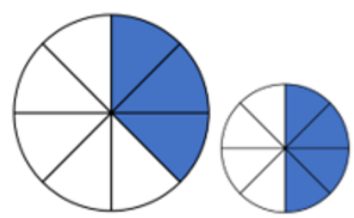
$\frac{2}{8}$ ☐ $\frac{2}{6}$

=   <   >

B.

321   123

C.

$\frac{3}{8}$ or $\frac{1}{2}$	

D.

5/3   5/4

2/5   4/5

3.5 Operations

This activity focuses on student thinking about operations, comparisons, and making sense of the unknown.

Operations: Students begin to develop meanings for operations in kindergarten (e.g., addition is putting together). As they progress, the numbers involved—and operational meanings—extend. Students should develop ways of thinking that enable them to connect operation meanings to everyday use of those operations. Operations should never be disconnected from meaning. Division of fractions, for example, is still a form of division and should connect to a meaning of division.

ITEM ALIGNMENT

CCSS: 3.OA.A.3

This item focuses on equal group multiplication situations. However, it also provides an opportunity to talk about equality and the meaning of multiplication.

THE CONVERSATION STARTER

Use the information to complete the task.

Marcus has 5 flowerpots. He will plant 8 flowers in each pot.

Move numbers and a symbol into the boxes to represent how to find the total number of flowers Marcus will plant.

☐ ☐ ☐ = ☐

+ − × 3 5 8 13 32 40 45

CONVERSATION PATHS (QUESTIONS ONLY)*

A. Problem Solving: Strategy

What word or words in the problem statement help you to know what mathematical operation to use?

Ignoring the answer boxes, what are different ways you could solve this?

- Could you use addition to represent the total number of flowers Marcus will plant?
- What would that look like?

B. Content: Equal Sign (Meaning)

What does the equal sign mean?

Could you answer the question if the boxes looked like this? What would you do?

Let's look at some other equations with unknowns.

$$3 \times 9 = \square$$

$$\square \times 6 = 3$$

$$4 \times 9 = \square \times 6$$

- What does the box, or unknown, *mean* in each equation?
- What does the equal sign *mean* in each equation? Are they different in some way?

C. Content: Operations (Multiplication)

Let's talk about multiplication in general. What does multiplication do for us?

- Can I multiply any two numbers?
- Can you think of a real-world context involving two quantities where multiplication does *not* make sense?

D. Content: Operations (Multiplication, Commutativity)

How are these alike? How are they different?

$$8 \times 5 \qquad 5 \times 8$$

- How could these represent two different situations with flowers and pots?

CONVERSATION PATHS (ANNOTATED)*

A. Problem Solving: Strategy

What word or words in the problem statement help you to know what mathematical operation to use?

> ***The problem states that there are 8 flowers "in each pot." The words "in each" indicate that there are 5 groups of 8 flowers, which indicates multiplication would be helpful.***
>
> Ignoring the answer boxes, what are different ways you could solve this?
>
> ***Addition is a possibility.***

- Could you use addition to represent the total number of flowers Marcus will plant?

  Theoretically, yes. However, the space doesn't allow for that.

- What would that look like?

  You could add 5 groups of 8, or 8 + 8 + 8 + 8 + 8.

B. Content: Equal Sign (Meaning)

What does the equal sign mean?

> ***The equal sign tells me that the quantities on both sides of it have the same value.***

Could you answer the question if the boxes looked like this? What would you do?

> ***Thinking about the equal sign as a statement of equality between the sides allows us to swap the sides in the original answer.***

Let's look at some other equations with unknowns.

$$3 \times 9 = \square$$

$$\square \times 6 = 3$$

$$4 \times 9 = \square \times 6$$

- What does the box, or unknown, *mean* in each equation?

  It represents a value we don't know. In the first statement, the box is equal in value to the product of 3 and 9. In the second statement, the box means the number of copies of 6 needed to reach 30. In the last statement, it represents the number of copies of 6 needed to equal the product of 4 and 9.

- What does the equal sign *mean* in each equation? Are they different in some way?

  In each of the equations, the equal sign is a statement that one side is equal in value to the other. For students, though, we want to be sure that they can flexibly interpret it as relational. An operational perspective, such as "the result when you add..." will lead to difficulty with the third equation.

C. Content: Operations (Multiplication)

Let's talk about multiplication in general. What does multiplication do for us?

It can tell us how many things we have total when we have m groups of size n each. It can "copy" to compare something that is n times the size of something else.

- Can I multiply any two numbers?

  Numbers? Yes. But sometimes in real life it might not make sense to do so.

- Can you think of a real-world context involving two quantities where multiplication does *not* make sense?

  For example, if there are 10 third graders and 10 fourth graders, it would not make sense to multiply 10 x 10.

D. Content: Operations (Multiplication, Commutativity)

How are these alike? How are they different?

$$8 \times 5 \qquad 5 \times 8$$

From a calculation standpoint they are both equal to 40, so in that way they are similar. From a contextual standpoint, they might mean different things. In this situation, for example, if the first expression represents 8 flowers in each of 5 pots, then the second expression would represent 5 flowers in each of 8 pots. Those are two different situations, but both have the same number of total flowers.

- How could these represent two different situations with flowers and pots?

  See above.

3.5 Shareables*

Use the information to complete the task.

Marcus has 5 flowerpots. He will plant 8 flowers in each pot.

Move numbers and a symbol into the boxes to represent how to find the total number of flowers Marcus will plant.

$\square\square\square = \square$

+	−	×	3	5	8	13	32	40	45

B.

$\square = \square\square\square$

$$3 \times 9 = \square$$

$$\square \times 6 = 3$$

$$4 \times 9 = \square \times 6$$

D.

$$8 \times 5 \qquad 5 \times 8$$

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