Examine Construct Validity of Computerized Adaptive Test in K-12 Assessments

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ABSTRACT

The purpose of this study is to investigate the effect of missing data in computerized adaptive tests (CAT) on test construct validity. The CAT method is now becoming more popular in educational assessment. However, conducting construct validity on CAT data has unique challenges for researchers because of the nature of missing data in the CAT. Unlike linear tests in which missing mechanisms that is defined as missing type can be regarded as missing at random, the CAT algorithm determines that missing CAT is not random. The study using simulation methods examined the effect of different missing data generated from different IRT models on recovery of internal structure of tests at both item and item cluster levels. Results show it is impossible to recover the CAT test internal structure by using items as observable variables, but by parceling items and using parcels as observable variables, the test internal structure can be recovered. Parceling has the effect of over fitting models.

INTRODUCTION

Student achievement as measured in K-12 achievement tests is an abstract attribute. The construct of achievement is theoretically defined and operationalized by a test. The construct of a test is a theoretical representation of the underlying traits, concepts, attributes, processes, or structures the test is designed to measure and directly relates to test validity (Cronbach, 1971; Messick, 1989). The validity of a test is the extent to which it is designed to measure, and according to the *Standards for Educational and Psychological Testing* (American Educational Research Association [AERA], American Psychological Association [APA], & National Council on Measurement in Education [NCME], 1999), validity is the most important consideration in test development and evaluation. Five sources of validity evidence specified in the *Standards* include: (a) test content, (b) response process, (c) internal structure, (d) relations to other variables, and (e) consequences of testing. The test validation process in K-12 assessments relies heavily on content validation procedures (Kane, 2006; Lissitz & Samuelsen, 2007), but it shouldn't diminish the need for multiple sources of evidence to establish internal test meaning, including theoretical components, even for educational tests (Embretson, 2007). Any source of validity evidence should be viewed as supporting a specific interpretation or use of test scores.

Currently in K-12 education, most state tests and large-scale standardized assessment programs provide all or part of five sources of validity evidence for the interpretation of achievement test results in their test technical manuals. Different statistical techniques have been used to provide evidence to establish valid inference. Because confirmatory factor analysis (CFA) deals with relationships among sets of underlying latent variables and a larger number of observable indicators at either item or item cluster levels, CFA is currently the most frequently used method to provide evidence on the internal structure of a test and addresses the question of whether the items or subtests measure the hypothesized latent variable(s). However, applying the CFA method to investigate internal structure of tests has two major practical challenges: using categorical observable variables and missing data.

For the first challenge, the choice of the level of an indicator in general factor analysis, has a significant effect on evaluating the construct. For example, the choice of item level indicators or item cluster/parcel indicators that sum or average item scores as observable variables in the CFA could impact model evaluation result (Bandalos, 2002; Bandalos & Finney, 2001; Hall, Snell & Foust, 1999; Little, Cunningham, Shahar & Widaman, 2002; Nasser & Wisenbaker, 2003). Most item level variables used in K-12 assessment are categorical variables that are binary/dichotomous responses from multiple-choice or griddable items, and polytomous responses from constructed response items, performance events, or innovative items. Through the use of item parcels/clusters/testlets, sub-tests level variables can be assumed to be continuous observable variables and also be used as such in most current practices. However, some psychometric concerns (Bandalos & Finney, 2001; Bollen &Lennox, 1991; Coanders, Satorra, & Saris, 1997; Hall, Snell, & Foust, 1999; Marsh & O'Neill, 1984; Shevlin, Miles, & Bunting, 1997) over item parceling include: (a) loss of information about the relative importance of individual items, (b) parceling of ordinal scales with undefined values, (c) limited range of latent variables and biased variance and covariance parameters, and (d) underestimate the relationships of latent variable due to limited reliability of the scale. When the sample size is small, the benefits of using parcels over items as observable indicators (Bandalos, 2002; Bandalos & Finney, 2001; Bentler, 2009; Cattell, 1974; Hau & Marsh, 2004; Marsh, Hau, Balla, & Grayson, 1998; Nasser & Wisenbaker, 2003; West, Finch, & Curran, 1995; Yang & Green, 2010a) include:

(a) having fewer free parameters to be estimated compared to the number of observations and improves model fit, (b) reducing the problems of non-normality, (c) not requiring data transformation, and (d) robust normal theory estimation, or distribution-free estimation. However, these benefits will diminish when items are unidimensional or have high item communalities, and sample sizes are large.

One interesting issue is that the effect of parceling on estimates of factor analysis parameters closely relates to the choice of item response theory (IRT) models that most large scale programs use to score, equate, and scale the tests. The IRT models can be equivalent to item factor analysis (IFA) within latent variable modeling framework (McDonald, 1999, 2000; Muthen & Asparouhov, 2002; Muthén & Muthén, 2006). The IFA model is factor analysis that uses items as observable variables. Instead of modeling the linear relationship between indictors and latent variable(s) as is done in CFA, nonlinear relationships between items and the latent variable set are modeled through link functions that link latent variables to categorical observable variables. The IFA with equal discrimination functions or factor loadings is equivalent to the Rasch model (Rasch, 1960) or the one-parameter IRT model (Hambleton & Swaminathan, 1985), in which item discrimination parameters are constant. Other dichotomous IRT models, such 2-parameter and 3-parameter IRT models, can model items with different discrimination parameters, which is equivalent to IFA with non-equal equal discrimination functions or factor loadings. Studies on the parceling effect on nonlinear factor analysis with non-equal discrimination functions or factor loadings conditions are very limited (Ferrando, 2009). Most studies on the parceling effect consider only the parceling of continuous indicators that have equal discrimination functions or factor loadings and under these conditions. It is not surprising that parceling has little impact on the relationship between indicators and latent variables (Alhija & Wisenbaker, 2006; Bandolas, 2002; Hau & Marsh, 2004) in linear cases. Overall, parceling items is currently a commonly used technique based upon theoretical rationales.

The second challenge of using CFA to conduct construct validity analysis is missing data. The reasons missing data exists in educational assessments are numerous; some reasons include: (1) student behaviors, such as students motivation, failing to attend, unwilling to answer, cheating in taking a test; (2) scoring, such as scoring mistakes; and (3) administration and operation, such as lost test booklets, scanning mistakes, bad weather, fire alarm. Other reasons are due to test design, such as the choice of linear tests vs. computerized adaptive tests (CAT). According to Rubin’s (1976) missing data mechanisms, educational data can be classified as missing completely at random (MCAR), missing at random (MAR), or missing not at random (MNAR). Within the latent variable modeling framework (Muthén, Asparouhov, Hunter, & Leuchter, 2011), if missingness is related to observed variables, then it can be MAR; if missingness is related to latent variables, such as student achievement ability, then it is MNAR and such missing data refer to non-ignorable missing data. The focus of this paper is on data missing due to test design, i.e., missing data in CAT. Because a CAT operates on an algorithm that selects items from an item bank to match a student’s provisional ability estimate, each student test event contains responses to a small subset of the item pool. Two additional features of CAT data compared to linear test data are: (1) restricted range of person ability for given items and (2) persons with different ability get different items. If a whole item bank is imagined as a linear test, then missingness in a CAT can be taken as due to item responses missing from the collection of test sessions. For example, for Reading and Mathematics of *Measure of Academic Progress* (MAP, NWEA, 2011) tests, typical missing rates are around 98% because the ratio of item pool size to test length is around 50 and data are very sparse. Figure 1 illustrates five missing designs. For simplification, all designs assume there is no person missing data. Design 1 as the baseline design shows no missing on item and response, and Design 2 shows missing on the response. Both Designs 1 and 2 represent responses from linear tests shown in Tables 1 and 2. Design 3 and 4 represent neither linear nor CAT cases exactly because there will be no missing item responses in linear tests and some persons will answer some items in CATs. These two designs reflect the fact that observable indicators are not complete and are sampled from the existing pool of observable indicators according to content requirements in most CAT situations. The difference between Design 3 and 4 is that Design 4 contains additional missing responses. Tables 3 and 4 show the data pattern for CAT as Design 5. Besides the missing rate attributed to the CAT algorithm, restricting ability range in data also has an impact on factor analysis because of the restricted of range of observation variables.

Nowadays, CAT is becoming more popular in educational assessment. Right now, Oregon, Delaware, and Idaho use CAT in their state assessments, and several other states (Georgia, Hawaii, Maryland, North Carolina, South Dakota, Utah, and Virginia) are in various stages of CAT development. As a matter of fact, one of the two consortia was created as part of the Race to the Top initiative. The SMARTER Balanced Assessment Consortium (SBAC), consisting of over half of the states, is committed to a computerized adaptive model because it represents a unique opportunity to create a large-scale assessment system that provides maximally accurate achievement results for each student (Race to the Top Assessment Program, 2010). There is an urgent need to gain understanding about assessing construct validity in a CAT in real operation. The purpose of this study is to investigate the effect of missing data in CAT on construct validity of a test.

METHOD

Almost all large-scale standardized K-12 testing programs use an IRT model in scoring, equating, and scaling. The internal structure of a test is often reported as the evidence related to construct validity based on factor analysis. Some test programs report the internal structure using items as observable indicators or variables in factor analysis, others use item testlets or parcels as observable indicators or variables in factor analysis, and some other reports use both. The theoretical framework of this study is to use a unifying approach that combines both linear and non-liner factor analyses so that the impact of missing data on factor structure of a test can be compared at both item and item parcel/testlet levels. The linear factor analysis models the relationship between latent variables and continuous observable variables. While nonlinear factor analysis models relationships among latent variables and categorical observable variables, the IRT model can be considered as a special case of nonlinear factor analysis.

1. Factor Model

1.1 Continuous observed variables

Confirmatory factor analysis (CFA) that describes the covariance among observed variables as a function of latent factors makes some assumptions. These assumptions include that unique factors are normally distributed or independent with normally distributed residuals, manifest indictors are continuous and conditionally normal distributed, and there is a linear relationship between observed and latent factors. When sub-content or goal scores from tests are used as manifest indictors, the distribution properties usually meet these assumptions. Let y_{ij}

denote the continuous observed variables for person i =1, …, N on variable $j=1,.., J$, then the relationship between manifest indictors y_{ij} and a common latent factor f can be modeled as,

$$y_{ij} = \tau_j + \lambda_j \xi_i + \varepsilon_{ij}\,, \tag{1}$$

where τ_j is an intercept and λ_j is a factor loading for item j, ξ_i is a common factor score for person i, and ε_{ij} is residual. For more complicated the model such as the bifactor model, the equation 1 can be expanded to

$$\mathbf{Y} = \boldsymbol{\tau} + \boldsymbol{\Lambda}_\mathbf{y}\boldsymbol{\xi} + \boldsymbol{\varepsilon} \tag{2}$$

where $\mathbf{Y}$ is a (N x 1) column vector of manifest indictors, $\boldsymbol{\tau}$ is a (N x 1) column vector of measurement intercepts, $\boldsymbol{\Lambda}_\mathbf{y}$ is a (N x k) matrix of factor loadings, $\boldsymbol{\xi}$ is a (k x 1) column vector of factors, and $\boldsymbol{\varepsilon}$ is a (N x 1) column vector of residuals. For examples, figure 2 depicts a common factor model that has four observed continuous variables (assume that $\boldsymbol{\tau}$ =0) and the model can be expressed in equation 3:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} [\xi] + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}. \tag{3}$$

Figure 3 illustrates the bifactor model that can be expressed in following equation:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} \lambda_{g1} & \lambda_{11} & 0 \\ \lambda_{g2} & \lambda_{12} & 0 \\ \lambda_{g3} & 0 & \lambda_{21} \\ \lambda_{g4} & 0 & \lambda_{22} \end{bmatrix} \begin{bmatrix} \xi_g \\ \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}. \tag{4}$$

1.2 Categorical observed variables

When items that are categorically scored (such as dichotomously scored multiple-choice items) are used as observed variables, the distribution properties assumed by the CFA are usually violated. The categorical confirmatory factor analysis (CCFA) or item factor analysis (IFA) methods are needed. Christoffersson (1975) defined a set of unobservable variables, y*, which follows the multiple common-factor model described in equation 2,

$$\mathbf{Y}^* = \boldsymbol{\tau} + \boldsymbol{\Lambda}_\mathbf{y}\boldsymbol{\xi} + \boldsymbol{\varepsilon}. \tag{5}$$

Christoffersson (1975) assumed that the binary response variables y_j for jth unique factor, can be defined as

$$y_j = \begin{cases} 1 & \text{if } \mathrm{y}_\mathrm{j}^* \geq \tau_j \\ 0 & \text{if } \mathrm{y}_\mathrm{j}^* < \tau_j \end{cases}. \tag{6}$$

As shown in Figure 4, latent response variable formulation defines a threshold τ on a continuous underlying y* variable. If y* follows a logistic distribution, the IFA for dichotomous items can be obtained by modifying equation 1 through logit links:

$$\text{logit}(y_{ij}) = -\tau_j + \lambda_j \xi_i, \tag{7}$$

The difference between logit and probit links is that the y* has a variance of 3.29 (SD = 1.7) for the logistic distribution and 1.0 for the normal distribution. From equations 1 and 7, factor loading λ_j represents a discrimination parameter, while threshold parameter τ_j is an easiness parameter for CFA and difficulty parameter for IFA.

2. Item Response Models (IRT) – Bifactor, 2PL, 1PL, and Rasch Models

First, let $y_{ij(k)}$ denote the dichotomous response for person i =1, …, N on item $j=1,.., J$, embedded within item group $k=1,...,K$, with constraint $\sum_{k=1}^{K} J_k = J$. Second, let **u** denotes the response vector of all responses. Then the overall probability of person i answering item j within item group k correctly is conditioned on k group-specific latent ability θ_k and a general latent ability θ_g in the bifactor model (Gibbons and Hedeker, 1992), which is a special case of two-parameter multidimensional logistic model (Reckase, 1985), can be shown in Equation 1.

$$P(\boldsymbol{y}|\boldsymbol{\theta})=\prod_{j=1}^{J} P(u_{ij(k)} = 1 \,|\theta_g, \theta_k) = \frac{1}{1+\exp[-D(a_{jg}\theta_g + a_{jk}\theta_k + d_j)]} \tag{8}$$

Where $\boldsymbol{\theta} = (\theta_g, \theta_1, \theta_2, \ldots, \theta_k, \ldots, \theta_K)$, a_{jg} is general latent ability (g) discrimination parameter for item j, a_{jk} is k group-specific latent ability discrimination parameter for item j and , $d_j = -b_j\sqrt{a_{jg}^2 + a_{jk}^2}$ is the multidimensional intercept parameter for item j and b_j is the difficulty parameter for item j in two-parameter unidimensional logistic model (2PL), and D=1.7 is a scaling constant. In a bifactor model, the general latent ability θ_g and the group-specific latent ability θ_k are orthogonal. When a_{jk} =0 and θ_k=0 (and simplifying θ_g=θ), equation 1 becomes the two-parameter item response model (2PL) where θ is person ability parameter, b_j is item difficulty parameter, and $\theta \sim$ N(0, 1),

$$P(\boldsymbol{y}|\theta) = \prod_{j=1}^{J} P(u_{ij} = 1\,|\theta) = \frac{1}{1+\exp[-Da_j(\theta - b_j)]}\ , \tag{9}$$

when a_j=1 and D=1, the equation 2 becomes Rasch model,

$$P(\boldsymbol{y}|\theta) = \prod_{j=1}^{J} P(u_{ij} = 1\,|\theta) = \frac{1}{1+\exp(b_j - \theta\ )}, \tag{10}$$

When a_j=1 and D=1.7, the equation 2 becomes one-parameter IRT model (1PL),

$$P(\boldsymbol{y}|\theta) = \prod_{j=1}^{J} P(u_{ij} = 1\,|\theta) = \frac{1}{1+\exp[D(b_j - \theta)]}\ . \tag{11}$$

3. Relationship between Factor Models and Item Response Models

The IRT models are equivalent to item level factor models within latent variable modeling framework (McDonald, 1999, 2000; Muthen & Asparouhov, 2002; Muthén & Muthén, 2006). Assume a single factor ξ with factor mean α and factor variance φ for binary items, and IRT latent variable θ has a standard normal distribution $\theta \sim N(0,1)$. Then $\xi = \alpha + \sqrt{\varphi}\theta$. For the logit link function,

$$P(\boldsymbol{u}|\theta) = \prod_{j=1}^{J} P(u_{ij} = 1|\theta) = \frac{1}{1+\exp(\tau_j - \lambda_j \xi)} = \frac{1}{1+\exp[-Da_j(\theta - b_j)]}. \quad (12)$$

Where,

$$a_j = \frac{\lambda_j \sqrt{\varphi}}{D} \text{ or } \lambda_j = \frac{Da_j}{\sqrt{\varphi}} \quad (13)$$

and

$$b_j = \frac{\tau_j - \lambda_j \alpha}{\lambda_j \sqrt{\varphi}} \text{ or } \tau_j = \lambda_j(\alpha + b_j\sqrt{\varphi}) \quad (14)$$

Figures 5 to 7 illustrate Rasch, 2PL, and bifactor models in IFA framework.

4. Research Design

In order to examine the impact of CAT missingness on test constructs under different conditions, the independent variables manipulated in the study are missing designs (5 designs), IRT models (Rasch, 2PL, and bifactor), observation levels (item and testlet), test length, testlet length, missing rate, and missing mechanisms. The dependent variables are fit statistics. In order to exam the models fit to data generated from different models, each of the 3 models data are calibrated using data from rest of two models.

The reason to choose a relatively high ratio (0.6 in this study compared to most CAT cases where ration is 0.05) of test length to size of item bank is that the major intention of this study is to investigate missing due to CAT design and the unique characteristics of data missindue to item difficulty range restriction caused by selection of item information to match provisional ability. This study attempts to separate the effects of data missing in linear test and missing in CAT. For example, in Design 3, the items are randomly selected from a bank and the missing mechanism is MCAR; in Design 5, items are not randomly selected from a bank and the missing mechanism is MNAR. By checking any difference in dependent variables between the two designs, the effect of missing data will be revealed. All CAT data were generated based on two item selection criteria: (1) maximum Fisher information method and (2) sub-content balanced method (Kingsbury & Zara, 1989).

Table 5 presents the research design. It is worth noting that although all data are simulated, no replication has been done because there is no intent to check calibration quality at the item level. All testlets are sums of items for a given sub-content. For the bifactor model in equation 8, item discrimination parameters for group-specific factors/latent variables are fixed and have logit values of 0.5, 1.0, 1.5, 2.0, and 2.5 for corresponding specific factors. The general factor is distributed as $\theta \sim N(0,1)$. All items are dichotomously scored items.

5. Parameter Distributions

Table 6 lists the parameter distribution used to generate response data for different models. All sample sizes are 10,000 persons.

6. Evaluation Criteria

The dependent variables are used to evaluate the data-model fit. Several well-known goodness-of-fit indexes (GOF) were used to evaluate model fit: the chi-square χ^2coefficient, the comparative fit index (CFI), the Tucker-Lewis Index (TLI), weighted root-mean-square residual (WRMR), the root mean square error of approximation (RMSEA) and the standardized root mean square error residual (SRMR). When interpreting a significant χ^2, sample size must be taken into account because significant χ^2 does not necessarily indicate a model misfit when the sample size is large.

Hu and Bentler (1999) recommended using combinations of GOF indices to obtain a robust evaluation of data-model fit in structural equation modeling. The cutoff criterion values they recommended were CFI > 0.95, TLI > 0.95, RMSEA < 0.06, and SRMR < 0.08. For WRMR, the cutoff value is WRMR<1.00 (Yu, 2002). However, Hu and Bentler offer cautions about the use of GOF indices, suggesting that these values should be treated as "rules of thumb" instead of rigid standards. Current practice seems to have incorporated these new guidelines without sufficient attention to the limitations. Moreover, some researchers (Beauducel & Wittmann, 2005; Fan & Sivo, 2005; Marsh, Hau, & Wen, 2004; Yuan, 2005) believe that these cutoff values are too rigorous and may have limited generalizability to the levels of misspecification experienced in typical practice. In general practice, a "good enough" or "rough guideline" approach for absolute fit indices and incremental fit indices (such as CFI, GFI, NFI, and TLI) have been quite commonly accepted (Lance, Butts, & Michels, 2006). Under the relaxed criteria, cutoff values should be above 0.90 (0.90 benchmark), and for fit indices based on residuals matrix (such as RMSEA and SRMR), values below 0.10 are usually accepted. In this study, two sets of criteria were used to evaluate model fit. The first set of criteria is Hu and Bentler's criteria and the second set of criteria is relaxed criteria.

7. Data Analysis

All response data of different IRT models were generated using SAS (SAS Institute Inc., 2008) and calibrated using Mplus (Muthen & Muthen, 2011). For all ordered categorical data, weighted least squares – mean, variance (WLSMV) estimators (Muthén & Muthén, 1998) were used and for all continuous data, maximum likelihood (ML) estimators were used. In theory, the Rasch model is a special case of 2PL, and 2PL is a special case of the bifactor model, so that comparison among three nested models is possible. However, this study did not examine this information. It does not compare these nested models. Even though Mplus provides statistics, such as χ^2 difference, Akaike information criterion (AIC) or Bayesian information criterion (BIC) for non-nested model comparison, these results are not reported in this study

RESULTS

Table 7 displays the summary of goodness-of-fit results across observation level, models, and test designs. Table 8 presents patterns of data model fit based on different sets of fit criteria across observation level, models, and test designs.

1. Item Level Results

The study aims is to investigate overall data model fit when items are used as observable indicators, not individual item parameter recovery. Results in Table 7 show that all model calibrations converged except for Design 5 at the item level. Table 8 shows, in general, that from simple (Rasch) to complex (bifactor) models, while the complex models fit data generated from complex models well: (1) models recover well for their own data except for the bifactor model based on criteria set one and (2) simple models do not fit well for data generated from more complex models. The intention of both Design 3 that uses randomly selected items from the item bank and Design 4 that adds missing responses to Design 3 data is to see the effect of both item and response missing data without the effects of an adaptive algorithm. Clearly, for both designs, the model fits the data very well and either item or response missing data has drastic impact on construct recovery, i.e. the estimated construct of test is very close to the true construct of test. The results imply that for IRT Rasch and 2PL models, it will be reasonable to use item level data to conduct factor analysis and provide construct validity evidence for fixed/linear form tests; for bifactor model, item level data fits not excellent but, still fit data when relaxed criteria are used.

However, this is not the case for CAT and design 5 results show that it is impossible to fit CAT data into factor models at the item level for given simulated data and this implies that it is meaningless to investigate construct validity of tests using items as indicators for CAT data. Considering the goodness of fit for Designs 3 and 4, the only reason for non-convergence occurring for Design 5 is caused by tne missing mechanism (MNAR) of CAT algorithm that restricts the ranges and variance of both ability and item difficulty, thus restricting covariance.

2. Testlet Level Results

From Tables 7 and 8, it appears that overall data-model fit indicess are substantially improved at the testlet level compared to those fort item level data. CAT data fit 2PL and bifactor models well based on relaxed fit criteria. Even for criteria set one, the bifactor model fit CAT data well but neither Rasch nor 2PL models fit data as well as the bifactor model. However, at the testlet level, models tend to over-fit data in which data from more complex models can be fitted well with the simpler model. For example, the Rasch model fits data generated from 2PL and bifactor models well. One potential explanation for the difference in fit between Rasch/2PL models and the bifactor model is the fact that both Rasch and 2PL are unidimensional models while the bifactor model is a multi-dimensional model. For a unidimensional model, the choice of items in testlets may have a negligible effect on model fit, while for multi-dimensional model, the interaction between choice of item in testlet and group-specific factors may influence the model fit. Because the bifactor model takes group specific factors into account in modeling, the fit may be improved.

DISCUSSION AND CONCLUSIONS

Construct validity evidence closely relates to statistical methods such as CFA and IFA that deal with internal structures of achievement tests. Until recently, researchers have paid virtually no attention to the problem of the test construct validity effects in computerized adaptive tests. In this study, at both item and testlet levels, the effects of data missing mechanisms in different test designs are investigated. First, at the item level, results show that both item and response missing data characterized as MCAR for linear tests have no differential effects on models fit across Designs 1 to 4. Both unidimensional models (Rasch and 2PL) fit data well but the multidimensional model (bifactor model) recoveres data poorly. No unidimensional and multidimensional models fit CAT data mainly because the CAT algorithm restricts the range of person ability and item difficulty. Our results suggest that it is impossible to recover the construct of CAT at the item level across models–even though recovering of construct of linear tests are reasonably good. Second, at the testlet level, we demonstrate that item parceling substantially improve models fit. At least for criteria set one, constructs of CAT can be recovered partially for unidimensional models (Rasch and 2PL) and fully for the multidimensional model (bifactor model). For both sets of criteria, constructs of linear tests under Design 1 to 4 can be well recovered. The study shows that parceling over-fits model.

Limitations of this study include: (1) small size of item bank and (2) replication. Because of time intensive calibrating the item bank for all three models (290 hours per model per condition if 250 items used), relatively small item bank size were chosen and this affects the missing rate in CAT data. All data studied are based on only one replication which may affect generalizability of conclusions. The future directions should include increasing the size of the item bank and more replications under different simulation conditions.

Table 1. No Missing Data (Item Responses) from a Linear Test with Test Length = 5 and Number of Person = 20

	Item						
	Sub-content $_1$		Sub-content 2			Sub-Total_1	Sub-Total_2
Person	I_1	I_2	II_1	II_2	II_3	RS1	RS2
P1	1	1	1	1	0	2	2
P2	1	1	1	0	0	2	1
P3	1	0	1	1	0	1	2
P4	1	1	0	1	0	2	1
P5	1	1	1	0	1	2	2
P6	1	1	0	0	0	2	0
P7	1	0	1	1	0	1	2
P8	1	1	0	1	0	2	1
P9	1	1	1	0	1	2	2
P10	1	1	0	0	0	2	0
P11	1	1	1	0	0	2	1
P12	1	0	1	1	0	1	2
P13	1	1	0	1	0	2	1
P14	0	1	1	0	1	1	2
P15	1	1	0	0	0	2	0
P16	1	0	1	1	0	1	2
P17	1	1	0	1	0	2	1
P18	1	1	1	1	0	2	2
P19	1	1	1	0	0	2	1
P20	1	0	1	1	0	1	2

Table 2. Missing Data (Item Responses) from a Linear Test with Test Length = 5 and Number of Person = 20

	Item						
	Sub-content $_1$		Sub-content 2			Sub-Total_1	Sub-Total_2
Person	I_1	I_2	II_1	II_2	II_3	RS1	RS2
P1	1	1	1	1	0	2	2
P2	1	.	1	0	.	1	1
P3	1	0	1	1	0	1	2
P4	1	1	0	1	0	2	1
P5	.	1	1	0	1	1	2
P6	1	1	0	0	0	2	0
P7	1	0	1	1	0	1	2
P8	1	1	0	1	0	2	1
P9	1	.	1	.	1	1	2
P10	1	1	0	0	0	2	0
P11	1	1	1	0	.	2	1
P12	1	0	1	1	0	1	2
P13	1	1	0	1	0	2	1
P14	0	1	.	0	1	1	2
P15	1	1	0	0	0	2	0
P16	.	0	1	.	0	1	1
P17	1	1	0	1	0	2	1
P18	1	1	1	1	0	2	2
P19	1	.	1	0	0	1	1
P20	1	0	1	1	0	1	2

Table 3. Missing Data Due to Test Design from a CAT Test with Test Length = 5 out of Item Bank Size= 30 and Number of Person = 20

	Item: Sub-content 1												Item: Sub-content 2																		Sub-Total$_1$	Sub-Total$_2$
Person	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}	I_{12}	II_1	II_2	II_3	II_4	II_5	II_6	II_7	II_8	II_9	II_{10}	II_{11}	II_{12}	II_{13}	II_{14}	II_{15}	II_{16}	II_{17}	II_{18}	RS1	RS2
P1																1	0	1	1	0											2	2
P2																									1	1	0	1	0		2	1
P3							1	0	1	1		0																			1	2
P4											1		0	0	1	0															2	1
P5																	1	1	0		1	0									2	2
P6			1	1	0	1	0																								2	0
P7																1	1	1	0	0											1	2
P8												1	0	1	1	0															2	1
P9																						1	0	0	1	1					2	2
P10			1		1	0	0	1																							2	0
P11										1		1	0	1	0																2	1
P12												1	1		0	1	0														1	2
P13																1		1	0	1	0										2	1
P14																			0	1	1	1		0							2	2
P15		1	0	1	0	0																									2	0
P16									1	1	0		1		0																1	2
P17														1	1		0	1	0												2	1
P18																	1	1		0	1	0									2	2
P19								1	1		0	1	0																		2	1
P20																						0	1	1	0		0				1	2

Table 4. Missing Data (Due to Test Design) Sorted by Person Ability (from low to high) and Item Difficulty (from easy to hard) from a CAT Test with Test Length = 5 out of Item Bank Size= 30 and Number of Person = 20 Based on Table 3

	Item Sub-content $_1$ + Sub-content $_2$																														Sub-Total$_1$	Sub-Total$_2$
Person	I_1	I_2	II_3	I_4	II_{14}	I_6	I_7	I_8	I_{12}	II_{13}	I_{11}	I_9	I_{10}	I_5	II_{15}	II_{16}	II_{17}	II_{18}	I_3	II_{26}	II_{27}	II_{28}	II_{29}	II_{24}	II_{25}	II_{19}	II_{20}	II_{21}	II_{22}	II_{30}	RS1	RS2
P15	1		0	1	0	0																									2	0
P6		1	1	0	1	0																									1	2
P10			1		1	0	0																								0	2
P4				0	1	1	1		0																						1	2
P5						1	0	1	1	0																					3	0
P6								1		0	0	1	0																		2	0
P7									1	1	0		1	0																	2	1
P8												1	0	1	1	0															2	1
P9													1	1	1	0	0														2	1
P10														1	0	0	1	1													1	2
P11															1		1	0	1	0											1	2
P12																	1	1		0	1	0									3	0
P13																	1		1	0	1	0									2	1
P14																		0	1	1	1		0								1	2
P15																		1	1	0		1		0							1	2
P16																			1	1		0	1	0							1	2
P17																				1	1		0	1	0						3	0
P18																					1	1		0	1	0					3	0
P19																						0	1	1	0		0				2	0
P20																									1	1	0	1		0	3	0

Table 5. Research Design

Design	Observation Level	IRT Model	No. Item in Bank	Test Length	Testlet Length	Missing Rate (%)		Missing Mechanism
						Item	Item Response	
1	Item	Rasch	100	100	25	0	0	
		2PL	100	100	25	0	0	
		Bifactor	100	100	25	0	0	
	Testlet	Rasch	100	100	25	0	0	
		2PL	100	100	25	0	0	
		Bifactor	100	100	25	0	0	
2	Item	Rasch	100	100	25	0	50	MCAR
		2PL	100	100	25	0	50	MCAR
		Bifactor	100	100	25	0	50	MCAR
	Testlet	Rasch	100	100	25	0	50	MCAR
		2PL	100	100	25	0	50	MCAR
		Bifactor	100	100	25	0	50	MCAR
3	Item	Rasch	100	40	10	60	0	MCAR
		2PL	100	40	10	60	0	MCAR
		Bifactor	100	40	10	60	0	MCAR
	Testlet	Rasch	100	40	10	60	0	MCAR
		2PL	100	40	10	60	0	MCAR
		Bifactor	100	40	10	60	0	MCAR
4	Item	Rasch	100	40	10	60	50	MCAR
		2PL	100	40	10	60	50	MCAR
		Bifactor	100	40	10	60	50	MCAR
	Testlet	Rasch	100	40	10	60	50	MCAR
		2PL	100	40	10	60	50	MCAR
		Bifactor	100	40	10	60	50	MCAR
5	Item	Rasch	100	40	10	0	60	MNAR
		2PL	100	40	10	0	60	MNAR
		Bifactor	100	40	10	0	60	MNAR
	Testlet	Rasch	100	40	10	0	60	MNAR
		2PL	100	40	10	0	60	MNAR
		Bifactor	100	40	10	0	60	MNAR

Table 6. Parameter Distributions of Models

Model	θ	θ_g	θ_k	a_j	a_{jg}	a_{jk}	b_j	Sample Size
Rasch	$N(0,1)$						$N(0,1)$	10000
2PL	$N(0,1)$			Log($N(0,1)$)			$N(0,1)$	10000
Bifactor		$N(0,1)$	$N(0,1)$		Log($N(0,1)$)	Fixed values		10000

Table 7. Summary of Goodness-of-Fit Indexes of Tests across Observation Level, Models, and Test Designs*

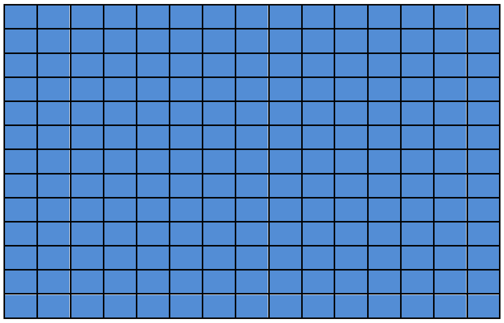
Observation Level	Calibration Model	GOF	Data Model: Rasch					2PL					Bifactor				
			Design														
			1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
Item	Rasch	p-Value	0.02	0.04	0.04	0.20	NC	0.00	0.00	0.00	0.00	NC	0.00	0.00	0.00	0.00	NC
		CFI	1.00	1.00	1.00	1.00	NC	0.49	0.65	0.592	0.65	NC	0.66	0.80	0.73	0.80	NC
		TLI	1.00	1.00	1.00	1.00	NC	0.68	0.69	0.664	0.67	NC	0.82	0.83	0.80	0.81	NC
		RMSEA	0.00	0.00	0.00	0.00	NC	0.13	0.07	0.129	0.06	NC	0.13	0.06	0.13	0.06	NC
		WRMR	1.01	1.01	1.01	0.99	NC	12.73	6.45	12.41	6.30	NC	12.73	6.40	12.29	6.14	NC
	2PL	p-Value	0.71	0.01	0.64	0.43	NC	0.00	0.26	0.00	0.16	NC	0.00	0.00	0.00	0.00	NC
		CFI	1.00	1.00	1.00	1.00	NC	1.00	1.00	1.00	1.00	NC	0.78	0.92	0.88	0.94	NC
		TLI	1.00	1.00	1.00	1.00	NC	1.00	1.00	1.00	1.00	NC	0.95	0.96	0.94	0.95	NC
		RMSEA	0.00	0.00	0.00	0.00	NC	0.01	0.00	0.01	0.00	NC	0.07	0.03	0.07	0.03	NC
		WRMR	0.85	0.94	0.82	0.89	NC	0.00	0.92	0.93	0.91	NC	5.91	3.08	5.70	2.97	NC
	Bifactor	p-Value	0.00	0.00	0.00	0.00	NC	0.00	0.00	0.00	0.00	NC	0.00	0.00	0.00	0.00	NC
		CFI	0.90	0.97	0.85	0.94	NC	0.99	1.00	0.95	0.98	NC	0.82	0.94	0.85	0.92	NC
		TLI	0.98	0.98	0.95	0.95	NC	1.00	1.00	0.98	0.98	NC	0.96	0.97	0.92	0.933	NC
		RMSEA	0.02	0.01	0.04	0.02	NC	0.01	0.00	0.04	0.02	NC	0.06	0.03	0.08	0.04	NC
		WRMR	1.96	1.30	3.1	1.79	NC	1.14	0.97	2.96	1.62	NC	5.18	2.71	6.46	3.31	NC
		p-Value	0.75	0.45	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
Testlet	Rasch	CFI	1.00	1.00	1.00	1.00	0.93	1.00	1.00	1.00	1.00	0.94	0.99	0.99	0.99	1.00	1.00
		TLI	1.00	1.00	1.00	1.00	0.80	1.00	1.00	1.00	1.00	0.81	0.96	0.98	0.98	1.00	0.99
		RMSEA	0.00	0.00	0.02	0.01	0.16	0.05	0.01	0.03	0.00	0.20	0.17	0.07	0.09	0.02	0.04
		SRMR	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.01	0.00	0.10	0.02	0.01	0.02	0.01	0.01
		p-Value	0.75	0.45	0.11	0.02	0.00	0.00	0.00	0.00	0.98	0.00	0.00	0.00	0.00	0.02	0.00
	2PL	CFI	1.00	1.00	1.00	1.00	0.93	1.00	1.00	1.00	1.00	0.94	0.99	0.99	0.99	1.00	1.00
		TLI	1.00	1.00	1.00	1.00	0.80	1.00	1.00	1.00	1.00	0.81	0.96	0.98	0.98	1.00	0.99
		RMSEA	0.00	0.00	0.02	0.01	0.16	0.05	0.01	0.03	0.00	0.20	0.17	0.07	0.09	0.02	0.04
		SRMR	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.01	0.00	0.10	0.02	0.01	0.02	0.01	0.01
		p-Value	0.75	0.45	0.00	0.11	0.00	0.00	0.20	0.00	0.04	38.55	0.00	0.00	0.00	0.00	0.00
	Bifactor	CFI	1.00	1.00	1.00	1.00	0.93	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	1.00
		TLI	1.00	1.00	1.00	1.00	0.80	1.00	1.00	1.00	1.00	0.99	0.96	0.98	0.98	1.00	0.99
		RMSEA	0.00	0.00	0.02	0.01	0.16	0.05	0.01	0.03	0.00	0.04	0.17	0.07	0.09	0.02	0.04
		SRMR	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.01	0.00	0.01	0.02	0.01	0.02	0.01	0.01

*Notes: NC represents no converge

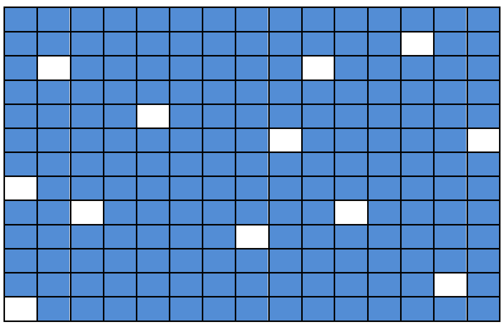
Table 8. Patterns of Data Model fit Based on Different Sets of Fit Criteria across Observation Level, Models, and Test Designs♣

			Criteria Set One♥															Criteria Set Two♠														
			Data Model															Data Model														
			Rasch					2PL					Bifactor					Rasch					2PL					Bifactor				
			Design															Design														
Observation Level	Calibration Model	GOF	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
Item	Rasch	p-Value																														
		CFI	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N	N	N	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N	N	N
		TLI	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N	N	N	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N	N	N
		RMSEA	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N	N	N	Y	Y	Y	Y	N	N	Y	N	Y	N	N	Y	N	Y	N
		WRMR	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N	N	N	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N	N	N
	2PL	p-Value																														
		CFI	Y	Y	Y	Y	N	Y	Y	Y	Y	N	N	N	N	N	N	N	Y	Y	Y	N	Y	Y	Y	Y	N	N	Y	N	Y	N
		TLI	Y	Y	Y	Y	N	Y	Y	Y	Y	N	N	Y	N	Y	N	Y	Y	Y	Y	N	Y	Y	Y	Y	N	Y	Y	Y	Y	N
		RMSEA	Y	Y	Y	Y	N	Y	Y	Y	Y	N	N	Y	N	Y	N	Y	Y	Y	Y	N	Y	Y	Y	Y	N	Y	Y	Y	Y	N
		WRMR	Y	Y	Y	Y	N	Y	Y	Y	Y	N	N	N	N	N	N	Y	Y	Y	Y	N	Y	Y	Y	Y	N	N	N	N	N	N
	Bifactor	p-Value																														
		CFI	N	Y	N	N	N	Y	Y	Y	Y	N	N	N	N	N	N	N	Y	N	Y	N	Y	Y	Y	Y	N	N	Y	N	Y	N
		TLI	Y	Y	N	N	N	Y	Y	Y	Y	N	N	Y	N	N	N	Y	Y	Y	Y	N	Y	Y	Y	Y	N	Y	Y	Y	Y	N
		RMSEA	Y	Y	Y	Y	N	Y	Y	Y	Y	N	N	Y	N	Y	N	Y	Y	Y	Y	N	Y	Y	Y	Y	N	Y	Y	Y	Y	N
		WRMR	N	N	N	N	N	N	Y	N	N	N	N	N	N	N	N	N	N	N	N	N	N	Y	N	N	N	N	N	N	N	N
Testlet	Rasch	p-Value																														
		CFI	Y	Y	Y	Y	N	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
		TLI	Y	Y	Y	Y	N	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
		RMSEA	Y	Y	Y	Y	N	Y	Y	Y	Y	N	N	N	N	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	N	Y	Y	Y	Y
		SRMR	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
	2PL	p-Value																														
		CFI	Y	Y	Y	Y	N	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
		TLI	Y	Y	Y	Y	N	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	N	Y	Y	Y	Y	Y
		RMSEA	Y	Y	Y	Y	N	Y	Y	Y	Y	N	N	N	N	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	N	N	Y	Y	Y	Y
		SRMR	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
	Bifactor	p-Value																														
		CFI	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
		TLI	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
		RMSEA	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	N	N	N	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	N	Y	Y	Y	Y
		SRMR	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

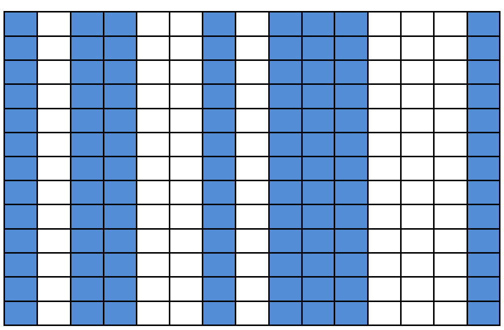
Notes: ♣Y represents fit; N represent not-fit. ♥: Criteria set one includes CFI > 0.95, TLI > 0.95, RMSEA < 0.06, SRMR < 0.08, and WRMR<1.00.
♠: Criteria set two includes CFI > 0.90, TLI > 0.90, RMSEA < 0.10, SRMR < 0.10, and WRMR<1.00.



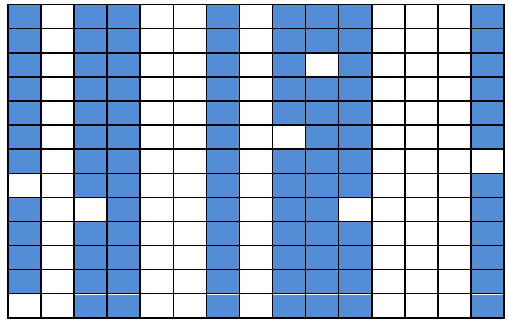
Design-1, Person: No Missing
Item: No Missing
Response: No Missing



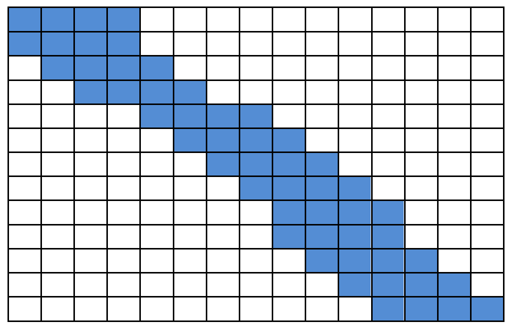
Design-2, Person: No Missing
Item: No Missing
Response: Missing MCAR



Design-3, Person: No Missing
Item: Missing MCAR
Response: No Missing



Design-4, Person: No Missing
Item: Missing MCAR
Response: Missing MCAR



Design-5, Person-Sorted: No Missing
Item-Sorted: No Missing
Response: Missing MNAR

Figure 1. Designs of Missing Data

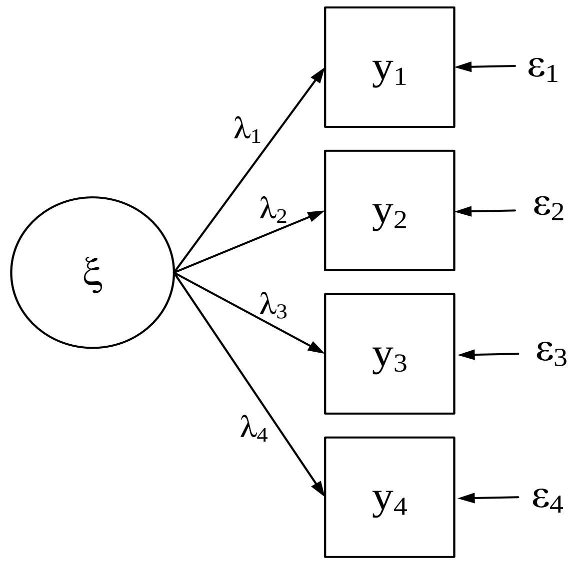


Figure 2. A Single Factor Model with Continuous Observed Variables

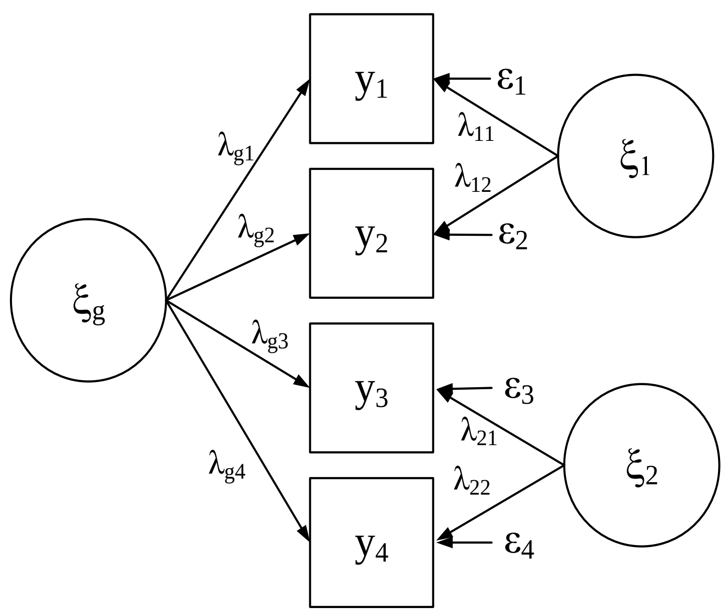


Figure 3. A Bifactor Model with Continuous Observed Variables

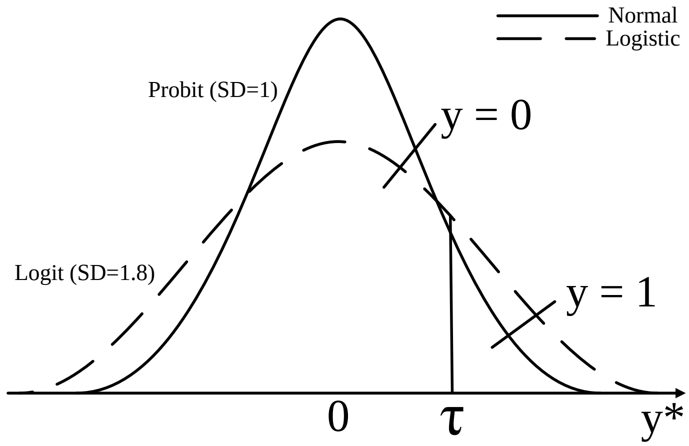


Figure 4. Relationship between y and y^*.

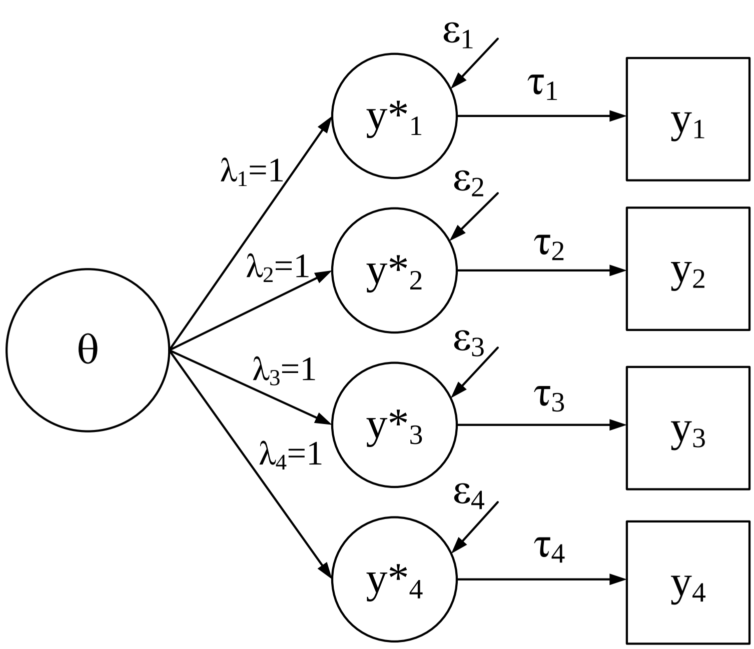


Figure 5. Rasch Model with Dichotomous Observed Variables

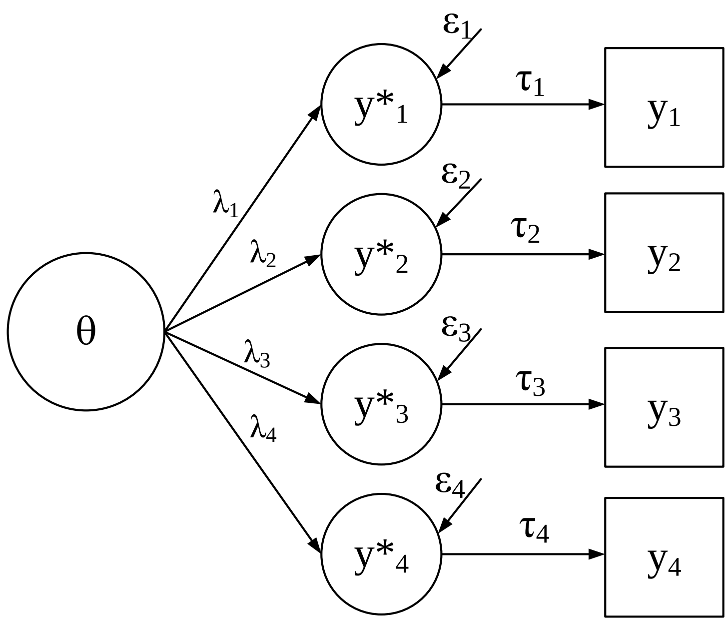


Figure 6. 2PL Model with Dichotomous Observed Variables

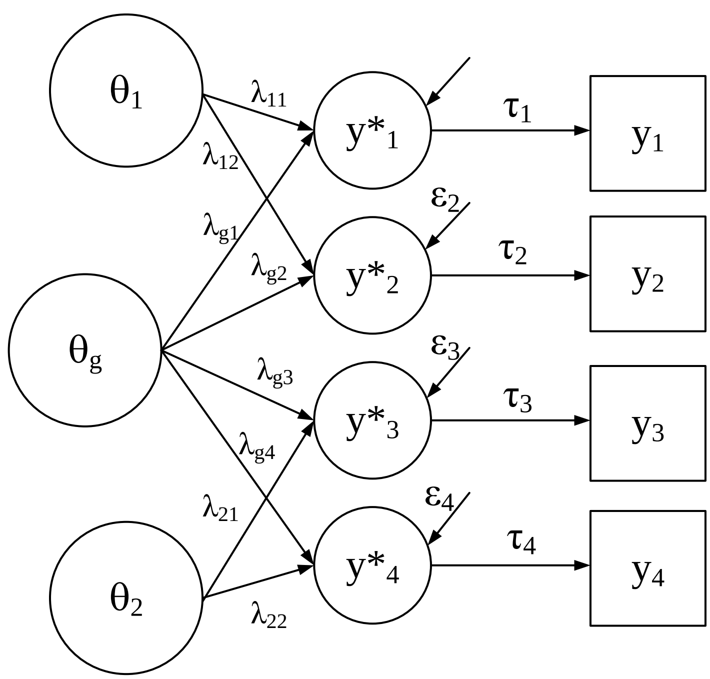


Figure 7. IRT-Bifactor Model with Dichotomous Observed Variables

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